DOCUMENT RESUME

ED 054 228 TM 000 810

AUTHOR	Joreskog, Karl G.; Van Thillo, Marielle
TITLE	New Rapid Algorithms for Factor Analysis by Unweighted Least Squares, Generalized Least Squares and Maximum Likelihood.
INSTITUTION	Educational Testing Service, Princeton, N.J.
REPORT NO	RM-71-5
PUB DATE	May 71
NOTE	78p.
EDRS PRICE	EDRS Price MF-$0.65 HC-$3.29
DESCRIPTORS	*Algorithms, *Computer Programs, Correlation, *Expectation, *Factor Analysis, Factor Structure, Mathematical Models, *Mathematics, Statistical Analysis
IDENTIFIERS	*FORTRAN IV G

ABSTRACT

A new basic algorithm is discussed that may be used to do factor analysis by any of these three methods: (1) unweighted least squares, (2) generalized least squares, or (3) maximum likelihood. (CK)

RESEARCH MEMORANDUM

RM-71-5

NEW RAPID ALGORITHMS FOR FACTOR ANALYSIS BY UNWEIGHTED LEAST SQUARES, GENERALIZED LEAST SQUARES AND MAXIMUM LIKELIHOOD

Karl G. Jöreskog and Marielle van Thillo

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Educational Testing Service
Princeton, New Jersey
May 1971

NEW RAPID ALGORITHMS FOR FACTOR ANALYSIS BY UNWEIGHTED LEAST SQUARES, GENERALIZED LEAST SQUARES AND MAXIMUM LIKELIHOOD

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1. Introduction

We shall describe a new basic algorithm that may be used to do factor analysis by any of the three methods

(i) unweighted least squares (ULS)

(ii) generalized least squares (GLS)

(iii) maximum likelihood (ML).

The ULS method produces solutions that are equivalent to those obtained by the iterated principal factor method and the minres method (see Harman, 1967, Chapters 8 and 9). The generalized least squares method is described by Jöreskog and Goldberger (1971). In the ML case, the new algorithm is simpler and faster than Jöreskog's (1967a,b) method UMLFA. It is similar to Clarke's (1970) algorithm but Heywood cases are handled in a simpler and more efficient way. Although the new algorithm handles ULS and GLS as well as ML, the computer program is shorter than UMLFA.

The GLS and ML methods are scale free. When multivariate normality is assumed, both GLS and ML yield estimates that are asymptotically efficient. Both GLS and ML require a positive definite variance-covariance matrix S or correlation matrix R ; ULS will work even on a matrix that is non-Gramian.

The model is the usual factor analysis model, which requires the population variance-covariance matrix or correlation matrix Σ of the observed variables to be of the form

$$\Sigma = \Lambda\Lambda' + \psi^2 \quad , \tag{1}$$

where Λ is a $p \times k$ matrix of factor loadings and ψ^2 is a $p \times p$ diagonal matrix of unique variances. The factors are assumed to be orthogonal.

The model (1) is fitted to the observed variance-covariance matrix S or to the corresponding correlation matrix R, by the minimization of a fitting function $F(\Lambda, \psi)$, which is different for each of the three methods. The minimization of $F(\Lambda, \psi)$ is done in two steps. First the conditional minimum of F for given ψ is found. This gives a function $f(\psi)$ which is then minimized numerically using the Newton-Raphson procedure. Function values and derivatives of f of first and second order are given in terms of the characteristic roots and vectors of a certain matrix A. In the GLS and ML cases a transformation from ψ_i to θ_i is made to obtain stable derivatives at $\psi_i = 0$. The basic formulas for ULS, GLS and ML are given in sections 2, 3 and 4 respectively.

2. Formulas for ULS

Fitting function: $F(\Lambda,\psi) = (1/2)\ \mathrm{tr}[(S - \Sigma)^2]$, (2)

$\Sigma = \Lambda\Lambda' + \psi^2$,

$\Lambda'\Lambda$ is assumed to be diagonal (3)

Matrix whose roots and vectors are computed: $A = S - \psi^2$ (4)

Characteristic roots: $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_p$

Corresponding orthonormal vectors: $\omega_1, \omega_2, \cdots, \omega_p$

Conditional solution for Λ for given ψ : $\tilde{\Lambda} = \Omega_1 \Gamma_1^{1/2}$, (5)

$\Gamma_1 = \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_k)$,

$\Omega_1 = [\omega_1 \omega_2 \cdots \omega_k]$

Function minimized by the Newton-Raphson method:

$$f(\psi) = (1/2) \sum_{m=k+1}^{p} \gamma_m^2 \qquad (6)$$

First order derivatives: $\partial f / \partial \psi_i = -2\psi_i \sum_{m=k+1}^{p} \gamma_m \omega_{im}^2$ (7)

Second order derivatives:

$$\partial^2 f / \partial\psi_i \partial\psi_j = 4[\psi_i \psi_j \sum_{m=k+1}^{p} \omega_{im}\omega_{jm} \sum_{n=1}^{k} \frac{\gamma_m + \gamma_n}{\gamma_m - \gamma_n} \omega_{in}\omega_{jn} + \delta_{ij} \sum_{m=k+1}^{p} (\psi_i^2 - \gamma_m/2)\omega_{im}^2] \qquad (8)$$

Approximate second order derivatives: $\partial^2 f / \partial\psi_i \partial\psi_j \doteq 4\psi_i \psi_j (\sum_{m=k+1}^{p} \omega_{im}\omega_{jm})^2$ (9)

3. Formulas for GLS

Fitting function: $F(\Lambda,\psi) = (1/2)\ \mathrm{tr}[(S^{-1}\Sigma - I)^2]$, (10)

$\Sigma = \Lambda\Lambda' + \psi^2$,

$\Lambda'\psi^{-2}\Lambda$ is assumed to be diagonal (11)

Matrix whose roots and vectors are computed: $A = \psi S^{-1}\psi$ (12)

Characteristic roots: $\gamma_1 \leq \gamma_2 \leq \cdots \leq \gamma_p$

Corresponding orthonormal vectors: $\omega_1, \omega_2, \cdots, \omega_p$

Conditional solution for Λ for given ψ : $\tilde{\Lambda} = \psi\Omega_1(\Gamma_1^{-1} - I)^{1/2}$, (13)

$\Gamma_1 = \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_k)$,

$\Omega_1 = [\omega_1\omega_2\cdots\omega_k]$

Transformation: $\psi_i = +\sqrt{e^{\theta_i}}$; $\theta_i = \log \psi_i^2$ (14)

Function minimized by the Newton-Raphson method:

$$f(\Theta) = (1/2) \sum_{m=k+1}^{p} (\gamma_m - 1)^2 \tag{15}$$

First order derivatives: $\partial f/\partial\theta_i = \sum_{m=k+1}^{p} (\gamma_m^2 - \gamma_m)\omega_{im}^2$ (16)

Second order derivatives:

$$\partial^2 f/\partial\theta_i\partial\theta_j = \delta_{ij}\partial f/\partial\theta_i + \sum_{m=k+1}^{p} \gamma_m\omega_{im}\omega_{jm} [\sum_{n=1}^{k} \gamma_n \frac{\gamma_m + \gamma_n - 2}{\gamma_m - \gamma_n} \omega_{in}\omega_{jn} + s^{ij}\psi_i\psi_j] \tag{17}$$

Approximate second order derivatives: $\partial^2 f/\partial\theta_i\partial\theta_j \doteq (\sum_{m=k+1}^{p} \omega_{im}\omega_{jm})^2$ (18)

4. Formulas for ML

Fitting function: $F(\Lambda,\psi) = \mathrm{tr}(\Sigma^{-1}S) - \log |\Sigma^{-1}S| - p$, (19)

$\Sigma = \Lambda\Lambda' + \psi^2$,

$\Lambda'\psi^{-2}\Lambda$ is assumed to be diagonal

Matrix of which roots and vectors are computed: $A = \psi S^{-1}\psi$

Characteristic roots: $\gamma_1 \leq \gamma_2 \leq \cdots \leq \gamma_p$

Corresponding orthonormal vectors: $\omega_1, \omega_2, \cdots, \omega_p$

Conditional solution for Λ for given ψ : $\tilde{\Lambda} = \psi\Omega_1(\Gamma_1^{-1} - I)^{1/2}$,

$\Gamma_1 = \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_k)$,

$\Omega_1 = [\omega_1 \omega_2 \cdots \omega_k]$

Transformation: $\psi_i = +\sqrt{e^{\theta_i}}$, $\theta_i = \log \psi_i^2$

Function minimized by the Newton-Raphson method:

$$f(\Theta) = \sum_{m=k+1}^{p} (\log \gamma_m + 1/\gamma_m - 1) \tag{20}$$

First order derivatives:
$$\partial f/\partial\theta_i = \sum_{m=k+1}^{p} (1 - 1/\gamma_m)\omega_{im}^2 \tag{21}$$

Second order derivatives:

$$\partial^2 f/\partial\theta_i\partial\theta_j = -\delta_{ij}\partial f/\partial\theta_i + \sum_{m=k+1}^{p} \omega_{im}\omega_{jm}\left[\sum_{n=1}^{k} \frac{\gamma_m + \gamma_n - 2}{\gamma_m - \gamma_n}\,\omega_{in}\omega_{jn} + \delta_{ij}\right] \tag{22}$$

Approximate second order derivatives:
$$\partial^2 f/\partial\theta_i\partial\theta_j \doteq \left(\sum_{m=k+1}^{p} \omega_{im}\omega_{jm}\right)^2 \tag{23}$$

5. Basic Minimization Algorithm

Let θ denote a column vector with elements $\theta_1, \theta_2, \cdots, \theta_p$ (GLS and ML) or $\psi_1, \psi_2, \cdots, \psi_p$ (ULS), and let h and H denote the column vector and matrix of corresponding derivatives $\partial g/\partial\theta$ and $\partial^2 g/\partial\theta\partial\theta'$, respectively. Let $\theta^{(s)}$ denote the value of θ in the s^{th} iteration and let $h^{(s)}$ and $H^{(s)}$ be the corresponding vector and matrix of first- and second-order derivatives. The iteration procedure may then be written

$$H^{(s)}\delta^{(s)} = h^{(s)} \quad , \tag{24}$$

$$\theta^{(s+1)} = \theta^{(s)} - \delta^{(s)} \quad , \tag{25}$$

where $\delta^{(s)}$ is a column vector of corrections determined by (24). The Newton-Raphson procedure is therefore easy to apply, the main computations in each iteration being the computation of the roots and vectors of A and the solution of the symmetric system (24). It has been found that the Newton-Raphson procedure is very efficient, generally requiring only a few iterations for convergence. The convergence criterion is that the largest absolute correction be less than a prescribed small number ϵ. The minimizing θ may be determined very accurately, if desired, by choosing ϵ very small.

In detail, the numerical method is as follows: the starting point $\theta^{(1)}$ is chosen as (see e.g., Jöreskog, 1963, eqs. 6.20 and 7.10 or Jöreskog, 1967, eq. 26),

$$\theta_i^{(1)} = \log[(1 - k/2p)/s^{ii}] \quad , \quad \psi_i^{(1)} = +\sqrt{[(1 - k/2p)/s^{ii}]} \quad ,$$

$$\psi_i^{(1)} = .6s_{ii} \text{ if } S \text{ is not positive definite,} \tag{26}$$

where s_{ii} and s^{ii} are the i^{th} diagonal element of S and S^{-1} respectively. The exact matrix H of second order derivatives given by (8), (17) or (22) may not be positive definite in the beginning. Therefore, the approximation E given by (9), (18) or (23) is used in the first iteration and for as long as the maximum absolute correction is greater than a given constant ϵ_E (see sec. 7). After that, H is used if it is positive definite. It has been found empirically that E gives good reductions in function values in the early iterations but is comparatively ineffective near the minimum, whereas H near the minimum is very effective.

In each iteration we compute the characteristic roots and vectors of A by the Householder transformation to tridiagonal form, the QR method for the roots of the tridiagonal matrix and inverse iteration for the vectors. This is probably the most efficient method available (see Wilkinson, 1965). The system of equations (24) is solved by the square root factorization $H = TT'$, where T is lower triangular. This shows at an early stage whether H is positive definite or not.

In Heywood cases, when one or more of the $\theta_i \to -\infty$, i.e., $\psi_i \to 0$, a slight modification of the Newton-Raphson procedure is necessary to achieve fast convergence. This is due to the fact that the search for the minimum is then along a "valley" and not in a quadratic region. For ML and GLS, when $\theta_i \to -\infty$, $\partial g/\partial\theta_i \to 0$ and $\partial^2 g/\partial\theta_i\partial\theta_j \to 0$, $j = 1,2,\ldots,p$, so that when θ_i is small the i^{th} element of h and the i^{th} row and column of H and E are also small. This tends to produce a "bad" correction vector δ and the function may increase instead of decrease. A simple and effective way to deal with this problem is to delete the i^{th}

equation in the system (24) and compute the corrections for all the other θ's from the reduced system. One then computes the correction for θ_i as

$$\delta_i = (\partial g/\partial\theta_i)/(\partial^2 g/\partial\theta_i^2) \quad . \tag{27}$$

This procedure will decrease θ_i slowly in the beginning but faster the more evident it is that θ_i is a Heywood variable. When θ_i has become less than $\log(\epsilon)$ it is not necessary to change θ_i any more unless $\partial f/\partial\theta_i$ is negative. For ULS, an analogous procedure is used. When ψ_i becomes less than $\sqrt{\epsilon}$, ψ_i is not changed unless $\partial f/\partial\psi_i$ is negative. Thus, the procedure corrects itself quickly if a variable is incorrectly taken as a Heywood variable.

6. The Program

In this section we describe briefly what the program does. Details about the input are given in section 7. For those users who feel too restricted in their choice of an input matrix, as provided by the program, the kernel of the program is available as a subroutine. The input and output parameters for that subroutine will be described in section 8.

The input data may be raw data from which the matrix to be analyzed is computed, or it may be a dispersion matrix, or it may be a correlation matrix or a correlation matrix followed by a vector of standard deviations. From these input matrices, variables may be selected to be included in the analysis, so that the matrices to be analyzed could be of smaller order than the input matrices. Variables may also be interchanged with one another. The matrices to be analyzed may be dispersion matrices or correlation matrices. The user has the option to read in a starting point for ψ or have the program define a starting point (see sec. 5). This can be useful if convergence is slow and the user runs out of computer time. From the intermediate results the last ψ can be read in as a new starting point and minimization can continue.

For the given matrix S to be analyzed of order p by p and a given lower bound k_L and a given upper bound k_U for the number of factors, the program performs a sequence of factor analyses by the ML, ULS or GLS method of estimation chosen by the user and outlined in the previous sections. One such analysis is done for each number of factors

$$k = k_L, k_L+1, \ldots, k_U \quad .$$

The output will consist of the title with parameter listing and the matrix to be analyzed. Then for each number of factors k the unrotated factor loadings, the unique variances and the varimax-rotated factor loadings are printed. For ML and GLS this is followed by χ_k^2 and the corresponding degrees of freedom d_k, the probability level, i.e., the probability of obtaining a larger value of χ^2 than that actually obtained given that the model and the assumptions hold, and Tucker and Lewis' (1970) reliability coefficient ρ_k, defined as follows

$$C_0 = N - 1 (1/6)(2p + 5)$$

$$\chi_0^2 = C_0[\sum_{i=1}^{p} \log s_{ii} - \log |S|]$$

$$d_0 = \frac{1}{2} p(p - 1)$$

$$M_0 = \chi_0^2/d_0$$

$$C_k = C_0 - (2/3)k$$

$$\chi_k^2 = C_k f_{min}$$

$$d_k = \frac{1}{2} [(p - k)^2 - (p + k)]$$

$$M_k = \chi_k^2/d_k$$

$$\rho_k = \frac{M_0 - M_k}{M_0 - 1}$$

Finally the latent roots and their first differences at the minimum and the matrix of residual correlations are printed. The user also has an option to

print intermediate results consisting of the value of the function and the vector ψ at each iteration. Examples of input and output can be found in Appendix B.

The following <u>limitations</u> are imposed on the program:

max. no. of variables after selection = 30

max. no. of variables before selection = 75

max. no. of factors = 30

storage requirements on the IBM 360/65 = 120K (K = 1024 bytes)

The program can easily be modified to allow for a larger number of variables and factors. Instructions on how to change the maximum number of variables and factors allowed by the program can be found in Appendix C. The program is written in FORTRAN IV-G and has been tested out on the IBM 360/65 at Educational Testing Service. Double precision is used in floating point arithmetic throughout the program. With minor changes the program should run on any computer with a FORTRAN IV compiler. In computers with a single word length of 36 bits or more, single precision is probably sufficient.

Although the program has been working satisfactorily for all data analyzed so far, no claim is made that it is free of error and no warranty is given as to the accuracy and functioning of the program.

7. Input Data

For each set of data to be analyzed, the input consists of the following:

a. Title card

b. Parameter card

c. Data matrix

d. Selection cards (optional)

e. Starting point (optional)

f. New data or a STOP card

The function and setup of each of the above quantities are described in general terms below. Illustrative examples are given in Appendix B.

a. Title Card

Whatever appears on this card will appear on the first page of the printed output. All 80 columns of the card are available to the user.

b. Parameter Card

All quantities on this card, except for the logical indicators, must be punched as integers right adjusted within the field.

cols. 1-5	number of observations N
cols. 6-10	order of data matrix (p_0), before selection of variables
cols. 11-15	lower bound for the number of factors k_L
cols. 16-20	upper bound for the number of factors k_U
cols. 21-25	maximum number of iterations allowed for each number of factors k
col. 31	logical variable which determines whether selection of variables from the data matrix is desired col. 31 = T, if selection of variables is wanted col. 31 = F, if no selection of variables is wanted

col. 32 — logical variable which determines whether a dispersion matrix or a correlation matrix is to be analyzed

col. 32 = T, if a dispersion matrix is to be analyzed

col. 32 = F, if a correlation matrix is to be analyzed

col. 41 — integer indicator which determines whether raw data, a dispersion matrix, a correlation matrix or a correlation matrix with standard deviations are read in to determine the matrix to be analyzed

col. 41 = 1, read in raw data

col. 41 = 2, read in a dispersion matrix

col. 41 = 3, read in a correlation matrix (followed by a vector of standard deviations if col. 32 is T)

col. 42 — integer indicator which determines which method of estimation is to be used

col. 42 = 1 for ULS

col. 42 = 2 for GLS

col. 42 = 3 for ML

col. 43 — integer indicator which determines whether intermediate results are to be printed

col. 43 = 0, if no intermediate results are to be printed

col. 43 = 1, if intermediate results are to be printed (see sec. 6)

col. 44	integer indicator which determines whether a starting point is defined by the program or is to be supplied by the user (see secs. 5 and 6) col. 44 = 0 , if a starting point is defined by the program col. 44 = 1 , if a starting point is read in as data
cols. 46-55	convergence criterion ϵ (see sec. 5). For reasonable results use $\epsilon \leq .005$.
cols. 56-65	ϵ_E ; if all elements of the correction vector are less than ϵ_E the exact second order derivatives are computed in the minimization algorithm, otherwise the approximate second order derivatives are used (see sec. 5). From our experience $\epsilon_E = .1$ seems reasonable.
cols. 66-70	logical tape (disk) number of scratch tape (disk) used for intermediate storage

c. Data Matrix

The data matrix is preceded by a format card, containing at most 80 columns, beginning with a left parenthesis and ending with a right parenthesis. The format must specify floating point numbers consistent with the way in which the elements of the matrix are punched. Users who are unfamiliar with FORTRAN are referred to a FORTRAN Manual where format rules are given.

The input matrix can be any one of the following:

If col. 41 = 1 on the parameter card an $N \times p$ matrix of raw data is read in, one row at a time, starting a new card for each row. The matrix is preceded by a format card as described above.

If col. 41 = 2 the lower triangular part of a dispersion matrix, including the diagonal, is read in. The matrix should be punched row-wise as one long vector, i.e., there is no need to go to a new card if a new row starts. Again the matrix should be preceded by a format card.

If col. 41 = 3 and col. 32 = F the lower triangular part of a correlation matrix, including the diagonal, is read in. The matrix should be punched row-wise as one long vector, and should be preceded by a format card.

If col. 41 = 3 and col. 32 = T the lower triangular part of a correlation matrix, including the diagonal, is read in. The matrix should be punched row-wise as one long vector, and should be preceded by a format card. This matrix is then followed by a format card and a row-vector of standard deviations.

d. Selection Cards (optional)

Omit if column 31 of the parameter card is F. Otherwise the first card will have an integer value p punched in columns 1-5, right adjusted within the field. This integer will specify the new order of the data matrix after selection of variables ($p \leq p_0$).

The next card will contain integers, right adjusted in five column fields (i.e., sixteen such values will fit on one card), specifying which columns (rows) are to be included. For example, if $p_0 = 6$, $p = 3$ and the first, second and fifth columns (rows) are to be excluded, this card

would have a 3 punched in column 5, a 4 punched in column 10 and a 6 punched in column 15.

Note that if $p = p_0$ there will be no reduction in the size of the data matrix but columns (rows) can be interchanged.

e. Starting Point (optional)

Omit if column 44 on the parameter card is zero. Otherwise read in a starting ψ vector punched according to a format of 5D15.7 (see sec. 6) for each number of factors k.

f. Stacked Data

In the preceding paragraphs we have described how each set of data should be set up. Any number of such sets of data may be stacked together and analyzed in one run. After the last set of data in the stack, there must be a card with the word STOP punched in columns 1-4.

8. Subroutine NWTRAP

As an alternative to the program the user can write his own main program in which he calls the minimization package with a CALL to NWTRAP. The following subroutines are called by NWTRAP and thus are part of the package: FCTGR, INCPSI, SØLVE, ISMSL, HFWLIN, TRIDI, EIGVEC and QRB.

The minimization package should be called with the following sequence of FØRTRAN statements:

```
      DØ 10 K = KL,KU
      CALL NWTRAP(P,K,I1,I2,I3,S,EPS,EPSE,MAXIT,A,E,X,Y,FO,DET)
      .
      .
      .
   10 CØNTINUE
```

The DØ loop runs from KL, the lower bound on the number of factors to KU, the upper bound on the number of factors.

Next follows a description of the parameters of NWTRAP:

Input Parameters

P	order of data matrix S
K	number of factors
I1	determines which method of estimation is to be used
	I1 = 1 for ULS
	I1 = 2 for GLS
	I1 = 3 for ML
I2	determines whether intermediate results are to be printed (see sec. 6)
	I2 = 0 , if <u>no</u> intermediate results are to be printed
	I2 = 1 , if intermediate results are to be printed

I3 — determines whether the starting point is defined by the program or read in by the user as data (see sec. 6)

$I3 = 0$, if the starting point is defined by the program (see sec. 5)

$I3 = 1$, a starting vector ψ is read in as data with a format of 5D15.7 for each number of factors K

S — data matrix, stored row-wise as a vector. Should be singly dimensioned in the calling program by at least $(P(P + 1))/2$.

EPS — convergence criterion ϵ (see sec. 5). For reasonable results use $\epsilon \leq .005$.

EPSE — if all elements of the correction vector are less than EPSE, the exact second order derivatives are used in the minimization algorithm, otherwise the approximate second order derivatives are used. From our experience $EPSE = .1$ seems reasonable.

MAXIT — maximum number of iterations allowed for each number of factors K . Program exits if this number is exceeded.

Output Parameters

A — matrix of unrotated factor loadings, stored row-wise as a vector. Should be singly subscripted in the calling program by at least $P \times K$.

E	dummy vector. Should be dimensioned in the calling program by at least $(P \times (P + 1))/2$.
X	vector of unique variances. Should be dimensioned in the calling program by at least P .
Y	vector of latent roots of A at the minimum. Should be dimensioned in the calling program by at least P .
FO	the value of the function at the minimum
DET	the determinant of the data matrix S

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Appendix A

Listing of the FORTRAN Program

```
C       PROGRAM UFABY3
        IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
        INTEGER P,P2
        REAL HEAD,HALT
        COMMON/LIT/OS,OR
        COMMON/MN/DF,DET,FO,P,K,KL,N,NT,IND,P2
        DIMENSION FMT(10),S(2850),A(2850),HEAD(20),YY(30),E(465),Y(30)
        DATA HALT/4HSTOP/,FMT/24H(5X,I5,5X,10F11.3)        /
        CALL ERRSET(208,256,-1,1)
      1 READ(5,100)HEAD
        IF(HEAD(1).EQ.HALT)CALL EXIT
        READ(5,200)N,P,KL,KU,MAXIT  ,OS,OR,INDA,IND,IO,IS,EPS,EPSE,NT
        WRITE(6,300)HEAD,N,P,KL,KU,MAXIT,OS,OR,INDA,IND,IO,IS,EPS,EPSE,NT
        P2=(P*(P+1))/2
        CALL REX(P,P2,N,S,A,INDA)
        CALL PMSL(P,P,S,FMT,24HMATRIX TO BE ANALYZED   ,1,6,1)
        GO TO (4,2,2),IND
      2 REWIND NT
        WRITE(NT)(S(I),I=1,P2)
      4 DO 5 I=KL,KU
        K=I
        DF=((P-K)**2-(P+K))/2
        IF(DF.LT.0)GO TO 10
        CALL NWTRAP(P,K,   IND,IO,IS,S,EPS,EPSE,MAXIT,A,E,YY,Y,FO,DET)
        CALL FINDT(YY,Y,A,E,S)
      5 CONTINUE
        GO TO 1
     10 WRITE(6,400)K
        GO TO 1
    100 FORMAT(20A4)
    200 FORMAT(5I5,5X,2L1,8X,4I1,1X,2F10.0,I5)
    300 FORMAT(1H1,20A4/'ON=',I5/'OP=',I5/'OKL=',I5/'OKU=',I5/'OMAXIT=',
       1I5/'OLOGICAL VARIABLES= ',2L1/'OINTEGER VARIABLES= ',4I1/'OEPS=',
       2F10.7/'OEPSE=',F10.7/'OLOGICAL SCRATCH TAPE (DISK) NUMBER=',I3)
    400 FORMAT(1H0,'ERROR EXIT - DEGREES OF FREEDOM FOR ',I3,' FACTORS IS
       1NEGATIVE')
        END
        SUBROUTINE REX(P,P2,N,S,E,IND)
        IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
        INTEGER P,P2
        COMMON/LIT/OS,OR
        DIMENSION S(1),E(1),Y(100),X(100),FMT(10)
C ******* DEFINE INPUT MATRIX (RAW DATA OR S OR R OR R AND ST.DEV.)
        READ(5,100) FMT
        GO TO (5,25,30),IND
C ******* COMPUTE S FROM RAW DATA
      5 L=0
        DO 10 I=1,P
        X(I)=0.D0
        DO 10 J=1,I
        L=L+1
        S(L)=0.D0
     10 CONTINUE
        DO 15 K=1,N
        READ(5,FMT)(Y(J),J=1,P)
        L=0
        DO 15 I=1,P
        X(I)=X(I)+Y(I)
        DO 15 J=1,I
        L=L+1
        S(L)=S(L)+Y(I)*Y(J)
```

```
   15 CONTINUE
      FN=1.D0/N
      L=0
      DO 20 I=1,P
      X(I)=X(I)*FN
      DO 20 J=1,I
      L=L+1
      S(L)=S(L)*FN-X(I)*X(J)
   20 CONTINUE
      IF(OR)GO TO 50
   21 L=0
      DO 22 I=1,P
      L=L+I
   22 Y(I)=1.D0/DSQRT(S(L))
      GO TO 35
C ******* READ IN S
   25 READ(5,FMT)(S(I),I=1,P2)
      IF(OR)GO TO 50
      GO TO 21
C ******* READ IN R
   30 READ(5,FMT)(S(I),I=1,P2)
      IF(.NOT.OR)GO TO 50
C ******* READ IN STANDARD DEVIATIONS
      READ(5,100)FMT
      READ(5,FMT)(Y(I),I=1,P)
   35 L=0
      DO 40 I=1,P
      DO 40 J=1,I
      L=L+1
      S(L)=S(L)*Y(I)*Y(J)
   40 CONTINUE
   50 IF(OS)CALL SELECT(P,P2,S,E)
  100 FORMAT(10A8)
      RETURN
      END
      SUBROUTINE SELECT(P,P2,S,E)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
      DIMENSION S(1),E(1),MM(100)
      DO 5 I=1,P2
      E(I)=S(I)
    5 CONTINUE
      READ(5,100)P
      READ(5,200)(MM(I),I=1,P)
      P2=(P*(P+1))/2
      L=0
      DO 15 I=1,P
      DO 15 J=1,I
      LI=MM(I)
      L=L+1
      LJ=MM(J)
      IF(LI.GT.LJ)GO TO 10
      LI=LJ
      LJ=MM(I)
   10 IJ=((LI-1)*LI)/2+LJ
      S(L)=E(IJ)
   15 CONTINUE
  100 FORMAT(I5)
  200 FORMAT(16I5)
```

```
      RETURN
      END
      SUBROUTINE FINOT(V,Y,A,E,S)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
      COMMON/LIT/OS,OR
      COMMON/MN/DF,DET,FO,P,K,KL,N,NT,IND,P2
      DIMENSION V(1),E(1),FMT(10),Y(1),S(1),A(1)
      DATA BLANK/8H  - -    /
      DATA FMT/24H(5X,I5,5X,10F11.3)      /
      GO TO (1,2,3),IND
    1 WRITE(6,100)K
      GO TO 6
    2 WRITE(6,200)K
      GO TO 4
    3 WRITE(6,300)K
    4 REWIND NT
      READ(NT)(S(I),I=1,P2)
    6 CALL PMSL(P,K,A,FMT,28HUNROTATED FACTOR LOADINGS    ,0,7,0)
      WRITE(6,400)(V(I),I=1,P)
      CALL VARMAX(P,K,A,V,S)
      CALL PMSL(P,K,A,FMT,32HVARIMAX-ROTATED FACTOR LOADINGS ,0,8,0)
      GO TO (25,5,5),IND
    5 IF(K.NE.KL)GO TO 15
      CO=N-1.DO-(2.DO*P+5.DO)/6.DO
      FMO=0.DO
      IF(.NOT.OR)GO TO 9
      L=0
      DO 8 I=1,P
      L=L+I
      FMO=FMO+DLOG(S(L))
    8 CONTINUE
    9 FMO=CO*(FMO-DLOG(DET))/(.5DO*P*(P-1.DO))
   15 CHSQ=(CO-2.DO*K/3.DO)*FO
      PROB=CHIPR(DF,CHSQ)
      RHOK=(FMO-CHSQ/DF)/(FMO-1.DO)
      NF=DF
      WRITE(6,500)NF,CHSQ,PROB,RHOK
      L=P
      DO 22 I=1,P
      V(I)=Y(L)
   22 L=L-1
      DO 23 I=1,P
      Y(I)=V(I)
   23 CONTINUE
   25 V(1)=BLANK
      DO 28 I=2,P
      V(I)=Y(I-1)-Y(I)
   28 CONTINUE
      WRITE(6,600)Y(1),V(1),(I,Y(I),V(I),I=2,P)
C *** COMPUTE RESIDUAL CORRELATIONS    ( S - LAMBDA*LAMBDA' )
      L=0
      DO 35 I=1,P
      I1=(I-1)*K
      JL=0
      DO 30 J=1,I
      L=L+1
      RHOK=0.DO
      DO 29 M=1,K
```

```
      IL=I1+M
      JL=JL+1
   29 RHOK=RHOK+A(IL)*A(JL)
      E(L)=S(L)-RHOK
   30 CONTINUE
      IF(E(L).LE.0.D0)E(L)=1.D0
      V(I)=1.D0/DSQRT(E(L))
   35 CONTINUE
      L=0
      DO 40 I=1,P
      DO 40 J=1,I
      L=L+1
   40 E(L)=E(L)*V(I)*V(J)
      CALL PMSL(P,P,E,FMT,24HRESIDUAL CORRELATIONS    ,1,6,1)
  100 FORMAT('1UNWEIGHTED LEAST SQUARES SOLUTION FOR ',I3,' FACTORS')
  200 FORMAT('1GENERALIZED LEAST SQUARES SOLUTION FOR ',I3,' FACTORS')
  300 FORMAT('1MAXIMUM LIKELIHOOD SOLUTION FOR ',I3,' FACTORS')
  400 FORMAT(1H0,10X,'UNIQUE VARIANCES'/1H0,(10F11.3))
  500 FORMAT(1H0,10X,'CHISQUARE WITH',I5,  ' DEGREES OF FREEDOM IS',
     1F16.4//11X,'PROBABILITY LEVEL IS ',F6.3//11X,34HTUCKER'S RELIABILI
     2TY COEFFICIENT=  ,F6.3)
  600 FORMAT(1H1,15X,'LATENT ROOTS',5X,'FIRST DIFFERENCES'// 10X,'1',
     1D16.4,11X,A8/(6X,I5,D16.4,D19.4))
      RETURN
      END
      SUBROUTINE VARMAX(NT,NF,A,X,S)
C     ORTHOGONAL ROTATION BY KAISER'S VARIMAX METHOD
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(1),X(1),S(1)
      AT=NT
      AF=NF
      CK=DSQRT(2.D0)*.5D0
      L=0
      I1=0
      DO 16 I=1,NT
      I1=I1+I
      X(I)=1.D0/DSQRT(S(I1)-X(I))
      DO 16 J=1,NF
      L=L+1
      A(L)=A(L)*X(I)
   16 CONTINUE
      NF1=NF-1
   22 DO 90 K1=1,NF1
      K11=K1+1
      DO 90 K2=K11,NF
      AA=0.D0
      BB=0.D0
      CC=0.D0
      DD=0.D0
      IK1=K1
      IK2=K2
      DO 40 I=1,NT
      U=(A(IK1)+A(IK2))*(A(IK1)-A(IK2))
      V=2.D0*A(IK1)*A(IK2)
      AA=AA+U
      BB=BB+V
      CC=CC+(U+V)*(U-V)
      DD=DD+U*V
      IK1=IK1+NF
```

```
      IK2=IK2+NF
   40 CONTINUE
      XNUM=2.D0*(DD-AA*BB/AT)
      XDEN=CC-(AA**2-BB**2)/AT
      ANUM=DABS(XNUM)
      ADEN=DABS(XDEN)
      IF(ANUM-ADEN)47,46,52
   46 IF(ANUM)57,90,57
   47 ATA=ANUM/ADEN
      IF(ATA-.0011600)75,49,49
   49 ACO=1.D0/DSQRT(1.D0+ATA**2)
      ASI=ATA*ACO
      GO TO 59
   52 ACOT=ADEN/ANUM
      IF(ACOT-.0011600)81,54,54
   54 ASI=1.D0/DSQRT(1.D0+ACOT**2)
      ACO=ACOT*ASI
      GO TO 59
   57 ACO=CK
      ASI=CK
   59 ACO=DSQRT((1.D0+ACO)*.5D0)
      ASI=ASI/(2.D0*ACO)
      ACO=DSQRT((1.D0+ACO)*.5D0)
      ASI=ASI/(2.D0*ACO)
      IF(XDEN)64,64,68
   64 XCO=ACO*CK
      XSI=ASI*CK
      ACO=XCO+XSI
      ASI=XCO-XSI
   68 CO=ACO
      IF(XNUM)72,72,69
   69 SI=ASI
      GO TO 84
   72 SI=-ASI
      GO TO 84
   75 IF(XDEN)76,79,79
   76 CO=CK
      SI=CK
      GO TO 84
   79 K=K-1
      GO TO 90
   81 ACO=0.D0
      ASI=1.D0
      GO TO 59
   84 K=(NF*(NF-1))/2
      IK1=K1
      IK2=K2
      DO 89 I=1,NT
      AA=A(IK1)
      BB=A(IK2)
      A(IK1)=AA*CO+BB*SI
      A(IK2)=-AA*SI+BB*CO
      IK1=IK1+NF
   89 IK2=IK2+NF
   90 CONTINUE
   92 IF(K)93,93,22
   93 L=0
      DO 95 I=1,NT
      X(I)=1.D0/X(I)
```

```
      DO 95 J=1,NF
      L=L+1
      A(L)=A(L)*X(I)
   95 CONTINUE
C ***** CHANGE SIGN OF COLUMNS WHERE WARRANTED
      DO 130 J=1,NF
      AA=0.D0
      IJ=J
      DO 110 I=1,NT
      BB=DABS(A(IJ))
      IF(BB.LE.AA) GO TO 110
      AA=BB
      CC=A(IJ)
      IJ=IJ+NF
  110 CONTINUE
      IF(CC.GE.0.D0)GO TO 130
      IJ=J
      DO 120 I=1,NT
      A(IJ)=-A(IJ)
  120 IJ=IJ+NF
  130 CONTINUE
      RETURN
      END
      END
      FUNCTION CHIPR(DF,CHSQ)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      A=.5*DF
      X=.5*CHSQ
      IF(X .GT. 0.) GO TO 100
      CHIPR=1.
      GO TO 170
  100 TERM=1.
      SUM=0.
      COFN=A
      IF(13.-X) 110,110,120
  110 IF(A-X) 140,140,120
  120 CON=1.
      FACT=-A
  130 TEMP=SUM
      SUM=SUM+TERM
      COFN=COFN+1.
      TERM=TERM*X/COFN
      IF(SUM-TEMP) 160,160,130
  140 CON=0.
      FACT=X
  150 TEMP=SUM
      SUM=SUM+TERM
      COFN=COFN-1.
      RATIO=COFN/X
      TERM=TERM*RATIO
      IF(SUM-TEMP) 160,160,150
  160 CHIPR=CON+DEXP(DLOG(SUM)-X+A*DLOG(X)-DLGAMA(A))/FACT
  170 RETURN
      END
      END
      SUBROUTINE NWTRAP(P,K,    IND,ID,IS,S,EPS,EPSE,MAXIT,A,E,YY,Y,FO,
     1DET)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
```

```
      COMMON/KERN/G(30),V(30),VB(30)       ,D2(30),S1(30),S2(30),S3(30),
     1EPSU,BND,IM(30),MOR,KP,MORE,MAXTRY,P2,KP1
      DIMENSION S(1),E(1),A(1),YY(1),Y(1)
      IF(IO.EQ.1)WRITE(6,300)
      MAXTRY=10
      EPSU=.05D0
      MORE=0
      KP=P-K
      KP1=KP+1
      IF(EPS.LE..005000001)GO TO 1
      EPS=.005
      WRITE(6,100)
    1 P2=(P*(P+1))/2
      GO TO (4,2,2),IND
    2 CALL ISMSL (P,S,S,Y,DET,.5D-12,IERR)
      BND=DLOG(EPS)
      IF(IERR.NE.0)CALL EXIT
C******GENERATE STARTING PSI
    4 F0=1.D10
      GO TO (5,10,10),IND
    5 CALL ISMSL (P,S,E,Y,DET,.5D-12,IERR)
      BND=DSQRT(EPS)
      IF(IERR.EQ.0)GO TO 10
      IF(IS.EQ.0)GO TO 7
    3 READ(5,500)(V(I),I=1,P)
      GO TO (15,12,12),IND
   12 DO 13 I=1,P
   13 V(I)=2.D0*DLOG(V(I))
      GO TO 15
    7 L=0
      DO 6 I=1,P
      L=L+I
      V(I)=.6*S(L)
    6 CONTINUE
      GO TO 15
   10 IF(IS.EQ.1)GO TO 3
      FT=1.D0-K/(2.D0*P)
      L=0
      DO 11 I=1,P
      L=L+I
      GO TO (8,9,9),IND
    8 V(I)=DSQRT(FT/E(L))
      GO TO 11
    9 V(I)=DLOG(FT/S(L))
   11 CONTINUE
   15 MOR=1
      ITER=0
   16 CALL FCTGR(P,K,S,A,E,YY,Y,F0,IND)
      ITER=ITER+1
      IF(IO.EQ.0)GO TO 25
      GO TO (20,22,22),IND
   20 WRITE(6,200)ITER,F0,(V(I),I=1,P)
      GO TO 25
   22 WRITE(6,200)ITER,F0,(YY(I),I=1,P)
   25 IF(ITER.LE.MAXIT  )GO TO 26
      WRITE(6,600)
      CALL EXIT
   26 IF (MOR.EQ.0) GO TO 45
C******COMPUTE NEW PSI
```

```
      CALL INCPSI(P,K,IND,A,E,S,YY,Y,EPSE,EPS)
      GO TO 16
C******COMPUTE LAMBDA AT THE MINIMUM
   45 L=0
      GO TO (50,55,55),IND
   50 DO 52 I=1,P
      DO 51 J=1,I
      L=L+1
   51 E(L)=S(L)
   52 E(L)=E(L)-YY(I)
      L=1
      GO TO 61
   55 DO 60 I=1,P
      DO 60 J=1,I
      L=L+1
      E(L)=YY(I)*YY(J)*S(L)
   60 CONTINUE
      L=-1
   61 CALL HFWLIN(P,P2, L,K,E,Y,A,G ,VB,S1,S2,S3)
   62 L=0
      DO 65 I=1,P
      DO 65 J=1,K
      L=L+1
      GO TO (63,64,64),IND
   63 A(L )=A(L )*DSQRT(Y(J))
      GO TO 65
   64 A(L )=YY(I)*A(L )*DSQRT(1.D0/Y(J)-1.D0)
   65 CONTINUE
      GO TO (80,66,66),IND
   66 DO 75 I=1,P
      YY(I)=YY(I)**2
   75 CONTINUE
   80 RETURN
  100 FORMAT(1H0,'SPECIFIED CONVERGENCE CRITERION TOO LARGE  - EPS SET E
     1QUAL TO .005 ( SEE WRITE-UP)')
  200 FORMAT(1H0,'ITER=',I4,5X,'F=     ',D15.7/15X,'PSI=   ',7D15.7/
     1(21X,7D15.7))
  300 FORMAT(1H1,'INTERMEDIATE RESULTS'/)
  500 FORMAT(5D15.7)
  600 FORMAT('0MAXIMUM NUMBER OF ITERATIONS EXCEEDED')
      END
      SUBROUTINE FCTGR(P,K,S,A,E,YY,Y,F0,IND)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
      COMMON/KERN/G(30),V(30),VB(30)      ,D2(30),S1(30),S2(30),S3(30),
     1EPSU,BND,IM(30),MOR,KP,MORE,MAXTRY,P2,KP1
      DIMENSION S(1),A(1),YY(1),Y(1),E(1)
      ITRY=0
C******CHOOSE METHOD OF ESTIMATION(I.E.ULS,GLS.OR,ML)
    2 GO TO (1,30,30),IND
C******DEFINE(S-PSI**2),STORED IN E
    1 DO 5 I=1,P2
      E(I)=S(I)
    5 CONTINUE
      L=0
      DO 10 I=1,P
      YY(I)=V(I)**2
      L=L+I
      E(L)=E(L)-YY(I)
```

```
   10 CONTINUE
C******COMPUTE ROOTS AND VECTORS OF (S-PSI**2)
      L=P
      IF(MORE.EQ.0)L=KP
      CALL HFWLIN (P,P2,-1,L ,E,Y,A,G ,D2,S1,S2,S3)
      F=0.00
      DO 15 I=1,KP
      F=F+Y(I)**2
   15 CONTINUE
      F=F* .5D0
      JJ=0
      DO 25 I=1,P
      SUM=0.D0
      DO 20 J=1,KP
      JJ=JJ+1
      SUM=SUM+Y(J)*A(JJ)**2
   20 CONTINUE
      G(I)=-2.D0*V(I)*SUM
      JJ=JJ+MORE
   25 CONTINUE
      GO TO 65
C******DEFINE (PSI*S**-1*PSI), STORED IN E
   30 L=0
      DO 35 I=1,P
      YY(I)=DSQRT(DEXP(V(I)))
      DO 35 J=1,I
      L=L+1
      E(L)=YY(I)*YY(J)*S(L)
   35 CONTINUE
C******COMPUTE ROOTS AND VECTORS OF (PSI*S**-1*PSI)
      L=P
      IF(MORE.EQ.0)L=KP
      CALL HFWLIN (P,P2,1,L,E,Y,A,G ,D2,S1,S2,S3)
      F=0.D0
      GO TO (85,36,45),INO
   36 DO 40 I=1,KP
      F=F+(Y(I)-1.D0)**2
      D2(I)=Y(I)**2-Y(I)
   40 CONTINUE
      F=F*.5D0
      GO TO 55
   45 DO 50 I=1,KP
      F=F+1.D0/Y(I)+DLOG(Y(I))
      D2(I)=1.D0-1.D0/Y(I)
   50 CONTINUE
      F=F-KP
C******COMPUTE FIRST ORDER DERIVATIVES
   55 JJ=0
      DO 60 I=1,P
      SUM=0.00
      DO 58 J=1,KP
      JJ=JJ+1
      SUM=SUM+D2(J)*A(JJ)**2
   58 CONTINUE
      G(I)=SUM
      JJ=JJ+MORE
   60 CONTINUE
   65 IF(FO-F.LT.0.D0) GO TO 72
      FO=F
```

```
      DO 66 I=1,P
      VB(I)=V(I)
   66 CONTINUE
      RETURN
   72 ITRY=ITRY+1
      IF(ITRY.LE.MAXTRY)GO TO 73
      ITER=ITER-1
      WRITE(6,100)ITER,F,I,FO,MAXTRY
      CALL EXIT
   73 DO 75 I=1,P
      V(I)=(V(I)+VB(I))*.5D0
   75 CONTINUE
      GO TO 2
   85 CALL EXIT
  100 FORMAT('OF(',I3,')=',D15.7,' IS GREATER THAN F(',I3,')=',D15.7,' E
     1VEN AFTER',I4,' SUCCESSIVE HALVINGS OF THE INTERVAL'/ 'OINCREASING
     2 MAXTRY IN SUBROUTINE NWTRAP MIGHT SOLVE THE PROBLEM')
      END
      SUBROUTINE INCPSI(P,K,IND,A,E,S,YY,Y,EPSE,EPS)
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      INTEGER P,P2
      COMMON/KERN/G(30),V(30),VB(30)       ,D2(30),S1(30),S2(30),S3(30),
     1EPSU,BND,IM(30),MOR,KP,MORE,MAXTRY,P2,KP1
      DIMENSION E(1),A(1),S(1),YY(1),Y(1)
C ******* COMPUTE EXACT SECOND ORDER DERIVATIVES
      KPP=KP
      IF(MORE.EQ.0)GO TO 80
      KPP=P
      DO 1 I=1,P2
    1 E(I)=0.D0
      GO TO (11,5,11),IND
C ******* COMPUTE EXACT SECOND ORDER DERIVATIVES
    5 DO 10 I=1,P
      LL=I
      S2(I)=DSQRT(Y(I))
      DO 10 J=1,P
      A(LL)=A(LL)*S2(I)
      LL=LL+P
   10 CONTINUE
   11 DO 20 M=1,KP
      T1=Y(M)
      T2=.5D0*T1
      GO TO (13,12,12),IND
   12 T1=T1  -2.D0
   13 DO 14 N=KP1,P
      S3(N)=(T1+Y(N))/(Y(M)-Y(N))
   14 CONTINUE
      L=0
      LL=M
      DO 20 I=1,P
      LJ=M
      I1=(I-1)*P
      DO 18J=1,I
      J1=(J-1)*P
      L=L+1
      SUM=0.D0
      DO 15 N=KP1,P
      IN=N+I1
      JN=N+J1
```

```
      SUM=SUM+S3(N)*A(IN)*A(JN)
   15 CONTINUE
      GO TO (17,23,16),IND
   16 E(L)=E(L)+A(LL)*A(LJ)*SUM
      GO TO 24
   17 E(L)=E(L)+A(LL)*A(LJ)*SUM *V(I)*V(J)
      GO TO 24
   23 E(L)=E(L)+A(LL)*A(LJ)*(SUM+YY(I)*YY(J)*S(L))
   24 LJ=LJ+P
   18 CONTINUE
      GO TO (22,21,19),IND
   22 E(L)=E(L)+A(LL)**2*(YY(I)-T2)
      GO TO 21
   19 E(L)=E(L)+A(LL)**2
   21 LL=LL+P
   20 CONTINUE
      L=0
      DO 27 I=1,P
   27 S3(I)=G(I)
      GO TO (25,35,40),IND
   25 DO 30 I=1,P
      DO 30 J=1,I
      L=L+1
   26 E(L)=E(L)*4.D0
   30 CONTINUE
      GO TO 50
   35 DO 38 I=1,P
      L=L+I
   38 E(L)=E(L)+G(I)
      GO TO 50
   40 DO 45 I=1,P
      L=L+I
   45 E(L)=E(L)-G(I)
   50 L=0
      IHM=0
      DO 75 I=1,P
      L=L+I
      IF(E(L).GT.EPSU )GO TO 75
      IHM=IHM+1
      S1(IHM)=E(L)
      S2(IHM)=G(I)
      IM(IHM)=I
      E(L)=1.D0
      I1=I-1
      IF(I1.EQ.0)GO TO 73
      DO 72 J=1,I1
      JN=L-J
   72 E(JN)=0.D0
   73 JN=L
      I1=P-1
      DO 74 J=I,I1
      JN=JN+J
   74 E(JN)=0.D0
   75 CONTINUE
      CALL SOLVE(P,E,G,.5D-12,IERR)
      IF(MORE.EQ.0) GO TO 95
      IF(IERR.EQ.0)GO TO 101
      DO 76 I=1,P
   76 G(I)=S3(I)
```

```
C ******* COMPUTE EXPECTED SECOND ORDER DERIVATIVES
   80 L=0
      DO 92 I=1,P
      I1=(I-1)*KPP
      DO 92 J=1,I
      J1=(J-1)*KPP
      SUM=0.D0
      DO 90 M=1,KP
      IN=M+I1
      JN=M+J1
      SUM=SUM+A(IN)*A(JN)
   90 CONTINUE
      L=L+1
      E(L)=SUM**2
   92 CONTINUE
      GO TO (93,50,50),IND
   93 L=0
      DO 94 I=1,P
      DO 94 J=1,I
      L=L+1
      E(L)=E(L)* V(I)* V(J)*4.D0
   94 CONTINUE
      GO TO 50
   95 IF(IERR.EQ.0)GO TO 101
      WRITE(6,500)
      CALL EXIT
  101 IF(IHM.EQ.0)GO TO 87
      DO 102 I=1,IHM
      L=IM(I)
      G(L)=0.D0
      IF(S1(I).LT.1.D-10)GO TO 102
      G(L)=S2(I)/S1(I)
  102 CONTINUE
   87 DO 97 I=1,P
      V(I)=VB(I)-G(I)
   97 CONTINUE
      DO 98 I=1,P
      IF(V(I).GT.BND    )GO TO 96
      V(I)=BND
      GO TO 98
   96 IF(DABS(G(I)).LT.EPSE)GO TO 98
      MORE=0
      RETURN
   98 CONTINUE
C******TEST FOR CONVERGENCE
      DO 105 I=1,P
      IF(V(I).LE.BND    )GO TO 105
      IF(DABS(G(I)).GT.EPS)GO TO 106
  105 CONTINUE
      MOR=0
      RETURN
  106 MORE=K
      RETURN
  500 FORMAT('0EXPECTED SECOND ORDER DERIVATIVES MATRIX IS NOT POSITIVE
     1DEFINITE'/' THIS SHOULD NEVER HAPPEN - CHECK YOUR INPUT DATA')
      END
      SUBROUTINE ISMSL(N,A,B,Y,D,EPS,IERR)
C ***** INVERT SYMMETRIC MATRIX STORED LINEARLY
C         N    ORDER OF MATRIX
```

```
C         A    MATRIX TO BE INVERTED,STORED LINEARLY,MUST BE GRAMIAN
C         B    A INVERSE,STORED AS A VECTOR
C         Y    INTERNAL DUMMY ARRAY,MUST BE DIMENSIONED IN CALLING PROGRAM BY N
C         D    DETERMINANT(A)
C         EPS  IF ANY PIVOTAL ELEMENT IS LESS THAN EPS,A IS CONSIDERED SINGULAR
C              AND CONTROL IS TRANSFERRED TO THE CALLING PROGRAM WITH IERR=1
C         IERR =0 IMPLIES A IS NON-SINGULAR
C ********************
      IMPLICIT REAL*8(A-H,P-Z),LOGICAL*1(O)
      DIMENSION A(1),B(1),Y(1)
      IERR=0
      NN=(N*(N+1))/2
      DO 5 I=1,NN
    5 B(I)=A(I)
      D=1.D0
      IF(N.EQ.1) GO TO 260
      DO 240 L=1,N
      F=B(1)
      IF(F.LT.EPS) GO TO 700
      D=D*F
      F=1.D0/F
      NA=1
      DO 210 I=1,N
      NA=NA+I-1
  210 Y(I)=B(NA)
      NA=0
      NB=1
      DO 220 I=2,N
      NB=NB+1
      H=Y(I)*F
      DO 220 J=2,I
      NB=NB+1
      NA=NA+1
  220 B(NA)=B(NB)-Y(J)*H
      DO 230 J=2,N
      NA=NA+1
  230 B(NA)=-Y(J)*F
  240 B(NN)=-F
      DO 250 I=1,NN
  250 B(I)=-B(I)
      RETURN
  260 L=1
      F=B(1)
      IF(F.LT.EPS) GO TO 700
      B(1)=1.D0/F
      D=F
      RETURN
  700 WRITE (6,1) L,F,D
      IERR=1
      RETURN
    1 FORMAT('0MATRIX IS NOT POSITIVE DEFINITE',I5,2D15.7)
      SUBROUTINE SOLVE(N,A,X,ERR,IND)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(1),X(1)
      IND=0
      IF(N.EQ.1)GO TO 4
      DO 2 J=2,N
      J1=J-1
      JD=J1+(J1*(J1-1))/2
```

```
      IF(A(JD).LT.ERR)GO TO 5
      Y=1.D0/A(JD)
      X(J1)=X(J1)*Y
      DO 2 K=J,N
      KJ=J1+(K*(K-1))/2
      T=A(KJ)*Y
      DO 1 L=K,N
      JJ=(L*(L-1))/2
      LK=K+JJ
      LJ=J1+JJ
    1 A(LK)=A(LK)-T*A(LJ)
      X(K)=X(K)-A(KJ)*X(J1)
    2 A(KJ)=T
      IF(A(KJ+1).LT.ERR)GO TO 5
      X(N)=X(N)/A(LK)
      KJ=N+1
      DO 3 J=2,N
      KL=KJ
      JJ=KL-J
      J1=J-1
      DO 3 K=1,J1
      KL=KL-1
      JD=JJ+(KL*(KL-1))/2
    3 X(JJ)=X(JJ)-X(KL)*A(JD)
      RETURN
    4 X(1)=X(1)/A(1)
      RETURN
    5 IND=1
      RETURN
      END
      SUBROUTINE HFWLIN(N,NT ,M,LV,A,E,B,D1,D2,S1,S2,S3)
C     FOR A GIVEN SYMMETRIC MATRIX A THIS SUBROUTINE USES HOUSEHOLDER'S
C     METHOD TO REDUCE THE MATRIX TO CODIAGONAL FORM, THE QR ALGORITHM
C     TO COMPUTE ALL EIGENVALUES AND WILKINSON'S METHOD TO CALCULATE
C     EIGENVECTORS CORRESPONDING TO A SPECIFIED NUMBER OF THE LARGEST
C     OR SMALLEST EIGENVALUES.
C ***** N =      THE ORDER OF THE INPUT MATRIX
C***** NT =     N*(N+1)/2
C ***** M =      1(-1) MEANS THE EIGENVALUES ARE TO BE IN DESCENDING(ASCENDING)
C *****          ORDER
C ***** LV =      THE NUMBER OF EIGENVECTORS WANTED
C ***** A =      THE GIVEN MATRIX,STORED AS A VECTOR,READING ROW-WISE AND NOT
C *****          INCLUDING THE UPPER TRIANGULAR PART.SHOULD BE DIMENSIONED IN THE
C *****          CALLING PROGRAM BY AT LEAST NT.A WILL BE DESTROYED UPON RETURN
C *****          TO THE CALLING PROGRAM
C ***** E =      THE VECTOR OF EIGENVALUES,SHOULD BE DIMENSIONED IN THE CALLING
C *****          PROGRAM BY AT LEAST N.
C ***** B =      THE MATRIX OF EIGENVECTORS,STORED AS A VECTOR,READING ROW-WISE.
C *****          SHOULD BE DIMENSIONED IN THE CALLING PROGRAM BY AT LEAST N*LV.
C *****          THE EIGENVECTORS ARE NORMALIZED SO THAT B'B=I .
C ***** D1,D2,S1,S2,S3   ARE USED INTERNALLY AND SHOULD BE DIMENSIONED IN THE
C *****          CALLING PROGRAM BY AT LEAST N.
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(1),E(1),B(1),D1(1),D2(1),S1(1),S2(1),S3(1)
      IF(N.EQ.1)GO TO 250
      VX=1.D+25
      IF(M.GT.0) VX=-VX
      CALL TRIDI (   N,A,D1,D2,S1,S2)
      URM=0.D0
```

```
      DO 210 I=1,N
      S1(I)=D1(I)
      S2(I)=D2(I)**2
  210 ORM=DMAX1(ORM,DABS(S1(I))+S2(I))
      ORM=ORM+1.D0
      CALL QRB (    N,E,S1,S2,ORM,1.D-24)
      DO 216 I=1,N
      VW=VX
      DO 214 J=I,N
      IF(E(J).LE.VW.AND.M.GT.0.OR.E(J).GE.VW.AND.M.LT.0) GO TO 214
      VW=E(J)
      L=J
  214 CONTINUE
      E(L)=E(I)
  216 E(I)=VW
      IF(LV.EQ.0) RETURN
      DO 240 I=1,LV
      EV=E(I)
      CALL EIGVEC (N,NT,A,D1,D2,EV,S1,S2,S3)
      L=I
      DO 240 J=1,N
      B(L)=S1(J)
  240 L=L+LV
      RETURN
  250 E(1)=A(1)
      B(1)=1.D0
      RETURN
      END
      SUBROUTINE QRB (    N,G,A,BQ,ORM,EPS)
C     COMP.J.,1963,6,99-101.
C     FRANCIS, J.G.F. THE QR TRANSFORMATION-PART II.
C     MATRIX BY THE QR METHOD. COMM. ACM,1965,8,217-218.
C     BUSINGER, P.A. ALGORITHM 253-EIGENVALUES OF A REAL SYMMETRIC
C     N - ORDER OF MATRIX
C     G - VECTOR OF EIGENVALUES
C     A - PRINCIPAL DIAGONAL
C     BQ - SQUARED SUBDIAGONAL
C     ORM - MATRIX NORM /COMPUTED IN TRIDI/
C     EPS - RELATIVE MACHINE PRECISION
      EPSQ=EPS*ORM
      DIMENSION A(1),BQ(1),G(1)
      IMPLICIT REAL*8(A-H,O-Z)
      UM=0.D0
      M=N
  200 IF(M.EQ.0) RETURN
      M1=M-1
      I=M1
      K=M1
      BQ(1)=0.D0
      IF(BQ(K+1).GT.EPSQ) GO TO 210
      G(M)=A(M)
      UM=0.D0
      M=K
      GO TO 200
  210 I=I-1
      IF(BQ(I+1).LE.EPSQ) GO TO 211
      K=I
      GO TO 210
  211 IF(K.NE.M1) GO TO 220
```

```
C      TREAT 2X2 BLOCK SEPARATELY
       UM=A(M1)*A(M)-BQ(M1+1)
       SQ1=A(M1)+A(M)
       VW=(A(M1)-A(M))**2+4.D0*BQ(M1+1)
       SQ2=DSQRT(VW)
       VW=SQ1+SQ2
       IF(SQ1.LT.0.D0)VW=SQ1-SQ2
       AMBDA=.5D0*VW
       G(M1)=AMBDA
       G(M)=UM/AMBDA
       UM=0.D0
       M=M-2
       GO TO 200
  220  AMBDA=0.D0
       VA=DABS(A(M)-UM)
       VB=.5D0*DABS(A(M))
       IF(VA.LT.VB) AMBDA=A(M)+.5D0*DSQRT(BQ(M1+1))
       UM=A(M)
       SQ1=0.D0
       SQ2=0.D0
       U=0.D0
       DO 240 I=K,M1
C      SHORTCUT SINGLE QR ITERATION
       GAMMA=A(I)-AMBDA-U
       IF(SQ1.EQ.1.D0)GO TO 221
       PQ=GAMMA**2/(1.D0-SQ1)
       GO TO 222
  221  PQ=(1.D0-SQ2)*BQ(I)
  222  T=PQ+BQ(I+1)
       BQ(I)=SQ1*T
       SQ2=SQ1
       SQ1=BQ(I+1)/T
       U=SQ1*(GAMMA+A(I+1)-AMBDA)
  240  A(I)=GAMMA+U+AMBDA
       GAMMA=A(M)-AMBDA-U
       IF(SQ1.EQ.1.D0) GO TO 241
       VW=GAMMA**2/(1.D0-SQ1)
       GO TO 242
  241  VW=(1.D0-SQ2)*BQ(M1+1)
  242  BQ(M1+1)=SQ1*VW
       A(M)=GAMMA+AMBDA
       GO TO 200
       END
       SUBROUTINE EIGVEC(LP,LPT,R,A,B,E,V,P,Q)
C      WILKINSON, J.H. THE CALCULATION OF EIGENVECTORS OF CODIAGONAL
C      MATRICES. COMP.J.,1958,1,90-96.
C      MATULA, D.W. SHARE PROGRAM SUBMITTAL, 1962 F2 BCHOW.
       IMPLICIT REAL*8(A-H,O-Z)
       DIMENSION R(1),A(1),B(1),V(1),P(1),Q(1)                   BCHOW186
C   SET UP SIMULTANEOUS EQUATIONS FOR EIGEN VECTOR WITH EIGEN VALUE E  BCHOW187
       X=A(1)-E                                                  BCHOW188
       Y=B(2)                                                    BCHOW189
       LP1=LP-1                                                  BCHOW190
       DO 10 I=1,LP1                                             BCHOW191
       IF(DABS(X)-DABS(B(I+1))) 4,6,8
    4  P(I)=B(I+1)                                               BCHOW193
       Q(I)=A(I+1)-E                                             BCHOW194
       V(I)=B(I+2)                                               BCHOW195
       Z=-X/P(I)                                                 BCHOW196
```

```
      X=Z*Q(I)+Y                                                        BCHOW197
      IF(LP1-I)5,10,5                                                   BCHOW198
    5 Y=Z*V(I)                                                          BCHOW199
      GO TO 10                                                          BCHOW200
    6 IF(X)8,7,8                                                        BCHOW201
    7 X=1.0D-10
    8 P(I)=X                                                            BCHOW203
      Q(I)=Y                                                            BCHOW204
      V(I)=0.D0
      X=A(I+1)-(B(I+1)/X*Y+E)                                           BCHOW206
      Y=B(I+2)                                                          BCHOW207
   10 CONTINUE                                                          BCHOW208
C  SOLVE SIMULTANEOUS EQUATIONS FOR EIGEN VECTOR OF TRI-DIAGONAL MATRIX BCHOW209
   20 IF(X) 21,25,21                                                    BCHOW210
   21 V(LP)=1.D0/X
   22 I=LP1                                                             BCHOW212
      V(I)=(1.D0-Q(I)*V(LP))/P(I)
      X=V(LP)**2+V(I)**2                                                BCHOW214
   25 I=I-1                                                             BCHOW215
      IF(I) 26,30,26                                                    BCHOW216
   26 V(I)=(1.D0-(Q(I)*V(I+1)+V(I)*V(I+2)))/P(I)
      X=X+V(I)**2                                                       BCHOW218
      GO TO 25                                                          BCHOW219
   28 V(LP)=1.0D10
      GO TO 22                                                          BCHOW221
   30 X=DSQRT(X)
      DO 31 I=1,LP                                                      BCHOW223
   31 V(I)=V(I)/X                                                       BCHOW224
C  TRANSFORM EIGEN VECTOR TO SOLUTION OF ORIGINAL MATRIX                BCHOW225
      IF(LP.EQ.2)RETURN
      K=LP                                                              BCHOW227
      J=LPT-1
      DO 44 N=3,LP
      J=J-K
      K=K-1
      L=J
      Y=0.D0
      DO 35 I=K,LP                                                      BCHOW232
      Y=Y+V(I)*R(L)
   35 L=L+I
      L=J
      DO 40 I=K,LP                                                      BCHOW235
      V(I)=V(I)-Y*R(L)
   40 L=L+I
   44 CONTINUE
      RETURN
       END                                                              BCHOW240
      SUBROUTINE TRIDI(    LP,P,A,B,W,Q)
C     WILKINSON,J.H. HOUSEHOLDER'S METHOD FOR THE SOLUTION OF THE
C     ALGEBRAIC EIGENPROBLEM. COMP.J.,1960,3,23-27.
C     MATULA, D.W. SHARE PROGRAM SUBMITTAL, 1962 F2 BCHOW.
C  TRI-DIAGONALIZATION SUBROUTINE       DWM        1517-UB              BCHOW033
C ***** LP =     ORDER OF THE INPUT MATRIX
C ***** R =      INPUT MATRIX,RETURNS WITH MODIFIED W MATRICES
C ***** A =      NEW DIAGONAL
C ***** B =      NEW FIRST OFF DIAGONAL
C ***** W,Q =    INTERNAL ARRAYS,MUST BE DIMENSIONED IN THE CALLING PROGRAM
C *****          BY AT LEAST LP
      IMPLICIT REAL*8(A-H,O-Z)
```

```
      DIMENSION R(1),A(1),B(1),Q(1),W(1)                                BCHOW039
      LP1=LP-1                                                          BCHOW040
      B(1)=0.D0
      IF(LP-2)99,65,15
   15 KL=0
      DO 51 K=2,LP1
      KL=KL+K
      KJ=K+1
      K1=K-1
C     CALCULATE AND STORE MODIFIED COLUMN MATRIX W                      BCHOW056
      SUM=0.D0
      L=KL
      DO 20 J=K,LP
      SUM=SUM+R(L)**2
      L=L+J
   20 CONTINUE
      S=DSQRT(SUM)
      B(K)=DSIGN(S,-R(KL))
      IF(SUM.LE.1.D-14) GO TO  51
      S=1.D0/S
      W(K)=DSQRT(DABS(R(KL))*S+1.D0)
      X=DSIGN(S/W(K),R(KL))
      R(KL)=W(K)                                                        BCHOW065
      JJ=KL+K
      DO 30 I=KJ,LP                                                     BCHOW066
      W(I)=X*R(JJ)                                                      BCHOW068
      R(JJ)=W(I)
      JJ=JJ+I
C     CALCULATE NEW R MATRIX WITH ROW K-1 NOW HAVING ZEROS OFF 2ND DIAGONALBCHOW070
   30 CONTINUE
      L=KL
      DO 35 J=K,LP                                                      BCHOW071
      JJ=J+1                                                            BCHOW072
      Q(J)=0.D0
      DO 33 I=K,J                                                       BCHOW075
      L=L+1
      Q(J)=Q(J)+R(L)*W(I)
   33 CONTINUE
      L1=L
      L=L+K1
      IF(JJ-LP)34,34,36                                                 BCHOW078
   34 DO 35 I=JJ,LP                                                     BCHOW079
      L1=L1+I-1
      Q(J)=Q(J)+R(L1)*W(I)
   35 CONTINUE
   36 X=0.D0
      DO 40 J=K,LP                                                      BCHOW083
   40 X=X+W(J)*Q(J)                                                     BCHOW084
      X=X*.5D0
      DO 45 I=K,LP                                                      BCHOW086
   45 Q(I)=X*W(I)-Q(I)                                                  BCHOW087
      LL=KL-K1
      DO 50 I=K,LP                                                      BCHOW090
      LL=LL+I
      L=LL
      DO 50 J=I,LP                                                      BCHOW092
      R(L)=R(L)+Q(I)*W(J)+Q(J)*W(I)
      L=L+J
   50 CONTINUE
```

```
   51 CONTINUE
C  SCRT OUTPUT                                                          BCHOW095
   65 L=0
      DO 60 I=1,LP                                                      BCHOW097
      L=L+I
      A(I)=R(L)                                                         BCHOW099
   60 CONTINUE
      LPP=L-1
      B(LP)=R(LPP)
   99 RETURN
       END                                                              BCHOW104
      SUBROUTINE PMSL(N,K,A,FMT,TEXT,LC,LT,IND)
C ***** PRINT MATRIX STORED LINEARLY
C           N,K    ORDER OF MATRIX,I.E. A(NXK)
C           A      MATRIX TO BE PRINTED
C           FMT    VARIABLE FORMAT WITH WHICH A IS PRINTED,SPECIFIED IN THE CALLING
C                  PROGRAM THROUGH DATA CARD OR THE LIKE
C           TEXT   HOLLERITH TITLE OF MATRIX,NUMBER OF CHARACTERS IN THIS TITLE
C                  SHOULD BE A MULTIPLE OF 4
C           LC     CARRIAGE CONTROL DIGIT (I.E. LC=1 IMPLIES NEW PAGE,LC=0 IMPLIES
C                  DOUBLE SPACE  ETC)
C           LT     NUMBER OF WORDS IN TEXT,I.E. NUMBER OF CHARACTERS IN TEXT
C                  DIVIDED BY 4,NOT TO BE EXCEEDED BY 20  (SEE DIMENSION)
C           IND    =0 IMPLIES PRINT FULL MATRIX
C                  OTHERWISE PRINT SYMMETRIC MATRIX,I.E. ONLY LOWER TRIANGULAR PART
C **********************************
      IMPLICIT REAL*8(A-H,P-Z)
      REAL TEXT,FMT
      DIMENSION A(1),TEXT(20),FMT(1)
      LO=1
      LL=1
    1 WRITE(6,11)LC,(TEXT(I),I=1,LT)
      L=MINO(LO+9,K)
      WRITE(6,12)(I,I=LO,L)
      IF(IND.EQ.0)GO TO 2
      LL=LO
    2 DO 4 I=LL,N
      IF(IND.EQ.0)GO TO 3
      LCX=(I*(I-1))/2
      LOW=LCX+LO
      LR=LCX+MINO(I,L)
      GO TO 4
    3 LCX=(I-1)*K
      LOW=LCX+LO
      LR=LCX+L
    4 WRITE(6,FMT)I,(A(J),J=LOW,LR)
      IF(L.EQ.K)RETURN
      LO=LO+10
      LC=0
      GO TO 1
   11 FORMAT(I1,10X,20A4)
   12 FORMAT(1H0,10X,10I11)
```

Appendix B

An Example of Input Data

We shall illustrate how input data should be set up by means of four sets of data. In all four cases Harman's correlation matrix of twenty-four psychological tests for 145 children is the input matrix. In the first set of data all twenty-four variables are to be analyzed with 4 and 5 common factors using the ML method of estimation. Intermediate output is requested.

The second set of data is as the first except that the ULS method of estimation is used and no intermediate output is to be printed.

The third set of data selects the first 13 variables from the input matrix to be analyzed with 4 common factors and using the ML method of estimation. No intermediate output is to be printed.

The last set of data analyzes the matrix of the third data set with the Heywood variable (the 11th variable) removed. Thus 12 variables are selected from the first thirteen and are analyzed with 3 and 4 common factors using the ML method of estimation. The 12 variables selected are also rearranged so that the variables appear in the following order:

13, 10, 12, 5, 4, 9, 1, 8, 7, 2, 6, 3

i.e., the 13th variable is now the first, the 10th variable is now the second, etc.

The next two pages show card by card how the data should be punched. One line corresponds to one card. For all sets of data MAXIT has been set to 30 and the scratch unit is 4.

The results obtained from these data follow on subsequent pages.

```
HARMAN'S 24 PSYCHOLOGICAL TESTS           ML
  145   24    4    5   30      FF       3310      .0005        .1    4
(16F5.0)
   1. .318   1. .403 .317   1. .468 .230 .305   1. .321 .285 .247 .227   1. .335
 .234 .268 .327 .622   1. .304 .157 .223 .335 .656 .722   1. .332 .157 .382 .391
 .578 .527 .619   1. .326 .195 .184 .325 .723 .714 .685 .532   1. .116 .057-.075
 .099 .311 .203 .246 .285 .170   1. .308 .150 .091 .110 .344 .353 .232 .300 .280
 .484   1. .314 .145 .140 .160 .215 .095 .181 .271 .113 .585 .428   1. .489 .239
 .321 .327 .344 .309 .345 .395 .280 .408 .535 .512   1. .125 .103 .177 .066 .280
 .292 .236 .252 .260 .172 .350 .131 .195   1. .238 .131 .065 .127 .229 .251 .172
 .175 .248 .154 .240 .173 .139 .370   1. .414 .272 .263 .322 .187 .291 .180 .296
 .242 .124 .314 .119 .281 .412 .325   1. .176 .005 .177 .187 .208 .273 .228 .255
 .274 .289 .362 .278 .194 .341 .345 .324   1. .368 .255 .211 .251 .263 .167 .159
 .250 .208 .317 .350 .349 .323 .201 .334 .344 .448   1. .270 .112 .312 .137 .190
 .251 .226 .274 .274 .190 .290 .110 .263 .206 .192 .258 .324 .358   1. .365 .292
 .297 .339 .398 .435 .451 .427 .446 .173 .202 .246 .241 .302 .272 .388 .262 .301
 .167   1. .369 .306 .165 .349 .318 .263 .314 .362 .266 .405 .399 .355 .425 .183
 .232 .348 .173 .357 .331 .413   1. .413 .232 .250 .380 .441 .386 .396 .357 .483
 .160 .304 .193 .279 .243 .246 .283 .273 .317 .342 .463 .374   1. .474 .348 .383
 .335 .435 .431 .405 .501 .504 .262 .251 .350 .382 .242 .256 .360 .287 .272 .303
 .509 .451 .503   1. .282 .211 .203 .248 .420 .433 .437 .388 .424 .531 .412 .414
 .358 .304 .165 .262 .326 .405 .374 .366 .448 .375 .434   1.
HARMAN'S 24 PSYCHOLOGICAL TESTS           ULS
  145   24    4    5   30      FF       3100      .0005        .1    4
(16F5.0)
   1. .318   1. .403 .317   1. .468 .230 .305   1. .321 .285 .247 .227   1. .335
 .234 .268 .327 .622   1. .304 .157 .223 .335 .656 .722   1. .332 .157 .382 .391
 .578 .527 .619   1. .326 .195 .184 .325 .723 .714 .685 .532   1. .116 .057-.075
 .099 .311 .203 .246 .285 .170   1. .308 .150 .091 .110 .344 .353 .232 .300 .280
 .484   1. .314 .145 .140 .160 .215 .095 .181 .271 .113 .585 .428   1. .489 .239
 .321 .327 .344 .309 .345 .395 .280 .408 .535 .512   1. .125 .103 .177 .066 .280
 .292 .236 .252 .260 .172 .350 .131 .195   1. .238 .131 .065 .127 .229 .251 .172
 .175 .248 .154 .240 .173 .139 .370   1. .414 .272 .263 .322 .187 .291 .180 .296
 .242 .124 .314 .119 .281 .412 .325   1. .176 .005 .177 .187 .208 .273 .228 .255
 .274 .289 .362 .278 .194 .341 .345 .324   1. .368 .255 .211 .251 .263 .167 .159
 .250 .208 .317 .350 .349 .323 .201 .334 .344 .448   1. .270 .112 .312 .137 .190
 .251 .226 .274 .274 .190 .290 .110 .263 .206 .192 .258 .324 .358   1. .365 .292
 .297 .339 .398 .435 .451 .427 .446 .173 .202 .246 .241 .302 .272 .388 .262 .301
 .167   1. .369 .306 .165 .349 .318 .263 .314 .362 .266 .405 .399 .355 .425 .183
 .232 .348 .173 .357 .331 .413   1. .413 .232 .250 .380 .441 .386 .396 .357 .483
 .160 .304 .193 .279 .243 .246 .283 .273 .317 .342 .463 .374   1. .474 .348 .383
 .335 .435 .431 .405 .501 .504 .262 .251 .350 .382 .242 .256 .360 .287 .272 .303
 .509 .451 .503   1. .282 .211 .203 .248 .420 .433 .437 .388 .424 .531 .412 .414
 .358 .304 .165 .262 .326 .405 .374 .366 .448 .375 .434   1.
HARMAN'S 13 PSYCHOLOGICAL TESTS           ML
  145   24    4    4   30      TF       3300      .0005        .1    4
(16F5.0)
   1. .318   1. .403 .317   1. .468 .230 .305   1. .321 .285 .247 .227   1. .335
 .234 .268 .327 .622   1. .304 .157 .223 .335 .656 .722   1. .332 .157 .382 .391
 .578 .527 .619   1. .326 .195 .184 .325 .723 .714 .685 .532   1. .116 .057-.075
 .099 .311 .203 .246 .285 .170   1. .308 .150 .091 .110 .344 .353 .232 .300 .280
 .484   1. .314 .145 .140 .160 .215 .095 .181 .271 .113 .585 .428   1. .489 .239
 .321 .327 .344 .309 .345 .395 .280 .408 .535 .512   1. .125 .103 .177 .066 .280
 .292 .236 .252 .260 .172 .350 .131 .195   1. .238 .131 .065 .127 .229 .251 .172
 .175 .248 .154 .240 .173 .139 .370   1. .414 .272 .263 .322 .187 .291 .180 .296
 .242 .124 .314 .119 .281 .412 .325   1. .176 .005 .177 .187 .208 .273 .228 .255
 .274 .289 .362 .278 .194 .341 .345 .324   1. .368 .255 .211 .251 .263 .167 .159
 .250 .208 .317 .350 .349 .323 .201 .334 .344 .448   1. .270 .112 .312 .137 .190
 .251 .226 .274 .274 .190 .290 .110 .263 .206 .192 .258 .324 .358   1. .365 .292
 .297 .339 .398 .435 .451 .427 .446 .173 .202 .246 .241 .302 .272 .388 .262 .301
 .167   1. .369 .306 .165 .349 .318 .263 .314 .362 .266 .405 .399 .355 .425 .183
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.232 .348 .173 .357 .331 .413   1. .413 .232 .250 .380 .441 .386 .396 .357 .483
.160 .304 .193 .279 .243 .246 .283 .273 .317 .342 .463 .374   1. .474 .348 .383
.335 .435 .431 .405 .501 .504 .262 .251 .350 .382 .242 .256 .360 .287 .272 .303
.509 .451 .503   1. .282 .211 .203 .248 .420 .433 .437 .388 .424 .531 .412 .414
.358 .304 .165 .262 .326 .405 .374 .366 .448 .375 .434   1.
  13
   1    2    3    4    5    6    7    8    9   10   11   12   13
HARMAN'S 12 PSYCHOLOGICAL TESTS          ML
 145   24    3    4   30    TF        3300      .0005        .1    4
(16F5.0)
  1. .318   1. .403 .317   1. .468 .230 .305   1. .321 .285 .247 .227   1. .335
.234 .268 .327 .622   1. .304 .157 .223 .335 .656 .722   1. .332 .157 .382 .391
.578 .527 .619   1. .326 .195 .184 .325 .723 .714 .685 .532   1. .116 .057-.075
.099 .311 .203 .246 .285 .170   1. .308 .150 .091 .110 .344 .353 .232 .300 .280
.484   1. .314 .145 .140 .160 .215 .095 .181 .271 .113 .585 .428   1. .489 .239
.321 .327 .344 .309 .345 .395 .280 .408 .535 .512   1. .125 .103 .177 .066 .280
.292 .236 .252 .260 .172 .350 .131 .195   1. .238 .131 .065 .127 .229 .251 .172
.175 .248 .154 .240 .173 .139 .370   1. .414 .272 .263 .322 .187 .291 .180 .296
.242 .124 .314 .119 .281 .412 .325   1. .176 .005 .177 .187 .208 .273 .228 .255
.274 .289 .362 .278 .194 .341 .345 .324   1. .368 .255 .211 .251 .263 .167 .159
.250 .208 .317 .350 .349 .323 .201 .334 .344 .448   1. .270 .112 .312 .137 .190
.251 .226 .274 .274 .190 .290 .110 .263 .206 .192 .258 .324 .358   1. .365 .292
.297 .339 .398 .435 .451 .427 .446 .173 .202 .246 .241 .302 .272 .388 .262 .301
.167   1. .369 .306 .165 .349 .318 .263 .314 .362 .266 .405 .399 .355 .425 .183
.232 .348 .173 .357 .331 .413   1. .413 .232 .250 .380 .441 .386 .396 .357 .483
.160 .304 .193 .279 .243 .246 .283 .273 .317 .342 .463 .374   1. .474 .348 .383
.335 .435 .431 .405 .501 .504 .262 .251 .350 .382 .242 .256 .360 .287 .272 .303
.509 .451 .503   1. .282 .211 .203 .248 .420 .433 .437 .388 .424 .531 .412 .414
.358 .304 .165 .262 .326 .405 .374 .366 .448 .375 .434   1.
  12
  13   10   12    5    4    9    1    8    7    2    6    3
STOP
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HARMAN'S 24 PSYCHOLOGICAL TESTS          ML
N=  145
P=   24
KL=    4
KU=    5
MAXIT=   30
LOGICAL VARIABLES= FF
INTEGER VARIABLES= 3310
EPS= 0.0005000
EPSE= 0.1000000
LOGICAL SCRATCH TAPE (DISK) NUMBER=  4
```

MATRIX TO BE ANALYZED

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	0.318	1.000								
3	0.493	0.317	1.000							
4	0.468	0.230	0.305	1.000						
5	0.321	0.285	0.247	0.227	1.000					
6	0.335	0.234	0.268	0.327	0.622	1.000				
7	0.304	0.157	0.223	0.335	0.656	0.722	1.000			
8	0.332	0.157	0.382	0.391	0.578	0.527	0.619	1.000		
9	0.326	0.195	0.184	0.325	0.723	0.714	0.685	0.532	1.000	
10	0.116	0.057	-0.075	0.099	0.311	0.203	0.246	0.285	0.170	1.000
11	0.308	0.150	0.091	0.110	0.344	0.353	0.232	0.300	0.280	0.484
12	0.314	0.145	0.140	0.160	0.215	0.095	0.181	0.271	0.113	0.585
13	0.489	0.239	0.321	0.327	0.344	0.309	0.345	0.395	0.280	0.408
14	0.125	0.103	0.177	0.066	0.280	0.292	0.236	0.252	0.260	0.172
15	0.238	0.131	0.065	0.127	0.229	0.251	0.172	0.175	0.248	0.154
16	0.414	0.272	0.263	0.322	0.187	0.291	0.180	0.296	0.242	0.124
17	0.176	0.005	0.177	0.187	0.208	0.273	0.228	0.255	0.274	0.289
18	0.368	0.255	0.211	0.251	0.263	0.167	0.159	0.250	0.208	0.317
19	0.270	0.112	0.312	0.137	0.190	0.251	0.226	0.274	0.274	0.190
20	0.365	0.292	0.297	0.339	0.398	0.435	0.451	0.427	0.446	0.173
21	0.369	0.306	0.165	0.349	0.318	0.263	0.314	0.362	0.266	0.405
22	0.413	0.232	0.250	0.380	0.441	0.386	0.396	0.357	0.483	0.160
23	0.474	0.348	0.383	0.335	0.435	0.431	0.405	0.501	0.504	0.262
24	0.282	0.211	0.203	0.248	0.420	0.433	0.437	0.388	0.424	0.531

MATRIX TO BE ANALYZED

	11	12	13	14	15	16	17	18	19	20
11	1.000									
12	0.428	1.000								
13	0.535	0.512	1.000							
14	0.350	0.131	0.195	1.000						
15	0.240	0.173	0.139	0.370	1.000					
16	0.314	0.119	0.281	0.412	0.325	1.000				
17	0.362	0.278	0.194	0.341	0.345	0.324	1.000			
18	0.350	0.349	0.323	0.201	0.334	0.344	0.448	1.000		
19	0.290	0.110	0.263	0.206	0.192	0.258	0.324	0.358	1.000	
20	0.202	0.246	0.241	0.302	0.272	0.389	0.262	0.301	0.167	1.000
21	0.399	0.355	0.425	0.183	0.232	0.349	0.173	0.357	0.331	0.413
22	0.304	0.193	0.279	0.243	0.246	0.283	0.273	0.317	0.342	0.463
23	0.251	0.350	0.382	0.242	0.256	0.360	0.287	0.272	0.303	0.509
24	0.412	0.414	0.358	0.304	0.165	0.262	0.326	0.405	0.374	0.366

MATRIX TO BE ANALYZED

	21	22	23	24
21	1.000			
22	0.374	1.000		
23	0.451	0.503	1.000	
24	0.443	0.375	0.434	1.000

INTERMEDIATE RESULTS

```
ITER=   1    F=    0.1973708D 01
             PSI=  0.6695372D 00  0.8012988D 00  0.7161767D 00  0.7359262D 00  0.5478115D 00  0.5445127D 00  0.5381649D 00
                   0.5323443D 00  0.5126799D 00  0.6212547D 00  0.6486282D 00  0.6516243D 00  0.6498442D 00  0.7668719D 00
                   0.8050684D 00  0.7237529D 00  0.7339121D 00  0.7148514D 00  0.7616699D 00  0.7011788D 00  0.6948178D 00
                   0.7104082D 00  0.6342621D 00  0.6588050D 00

ITER=   2    F=    0.1717499D 01
             PSI=  0.6773125D 00  0.8890447D 00  0.8220106D 00  0.8111837D 00  0.5987694D 00  0.5649063D 00  0.5291028D 00
                   0.7012642D 00  0.5007284D 00  0.5361959D 00  0.7385593D 00  0.6564233D 00  0.6996297D 00  0.8136112D 00
                   0.8378412D 00  0.7436853D 00  0.7692774D 00  0.7725429D 00  0.8869384D 00  0.7711841D 00  0.7700830D 00
                   0.7789265D 00  0.7067444D 00  0.7231099D 00

ITER=   3    F=    0.1711112D 01
             PSI=  0.6637980D 00  0.8824492D 00  0.8049010D 00  0.8056633D 00  0.5922127D 00  0.5573563D 00  0.5327706D 00
                   0.6965713D 00  0.5075525D 00  0.5033638D 00  0.7390441D 00  0.6568816D 00  0.6986178D 00  0.8052307D 00
                   0.8341358D 00  0.7394329D 00  0.7724677D 00  0.7692933D 00  0.8729829D 00  0.7681030D 00  0.7638858D 00
                   0.7748646D 00  0.7045731D 00  0.7096049D 00

ITER=   4    F=    0.1710842D 01
             PSI=  0.6624246D 00  0.8831198D 00  0.8030295D 00  0.8065156D 00  0.5933544D 00  0.5582383D 00  0.5314280D 00
                   0.6967135D 00  0.5064482D 00  0.4932306D 00  0.7412843D 00  0.6584495D 00  0.6998659D 00  0.8037167D 00
                   0.8344472D 00  0.7402953D 00  0.7732623D 00  0.7698190D 00  0.8726947D 00  0.7689338D 00  0.7637106D 00
                   0.7752680D 00  0.7050339D 00  0.7076906D 00

ITER=   5    F=    0.1710821D 01
             PSI=  0.6621638D 00  0.8832289D 00  0.8021929D 00  0.8069802D 00  0.5933004D 00  0.5581273D 00  0.5316019D 00
                   0.6965785D 00  0.5065494D 00  0.4896464D 00  0.7422758D 00  0.6595946D 00  0.7005154D 00  0.8037265D 00
                   0.8342561D 00  0.7410087D 00  0.7734042D 00  0.7698349D 00  0.8726417D 00  0.7691675D 00  0.7634811D 00
                   0.7752598D 00  0.7051682D 00  0.7069368D 00

ITER=   6    F=    0.1710821D 01
             PSI=  0.6621666D 00  0.8832292D 00  0.8021943D 00  0.8069813D 00  0.5933005D 00  0.5581276D 00  0.5316027D 00
                   0.6966785D 00  0.5065487D 00  0.4895842D 00  0.7422800D 00  0.6596047D 00  0.7005202D 00  0.8037259D 00
                   0.8342656D 00  0.7410119D 00  0.7734036D 00  0.7698353D 00  0.8726416D 00  0.7691681D 00  0.7634810D 00
                   0.7752599D 00  0.7051682D 00  0.7069409D 00
```

MAXIMUM LIKELIHOOD SOLUTION FOR 4 FACTORS

UNROTATED FACTOR LOADINGS

	1	2	3	4
1	-0.553	0.044	0.454	-0.218
2	-0.344	-0.010	0.289	-0.134
3	-0.377	-0.111	0.421	-0.158
4	-0.465	-0.071	0.298	-0.198
5	-0.741	-0.225	-0.217	-0.038
6	-0.737	-0.347	-0.145	0.060
7	-0.738	-0.325	-0.242	-0.096
8	-0.696	-0.121	-0.033	-0.120
9	-0.749	-0.391	-0.160	0.060
10	-0.486	0.617	-0.378	-0.014
11	-0.540	0.370	-0.039	0.138
12	-0.447	0.572	-0.040	-0.191
13	-0.579	0.307	0.117	-0.258
14	-0.404	0.045	0.082	0.427
15	-0.365	0.071	0.162	0.374
16	-0.452	0.072	0.419	0.256
17	-0.438	0.190	0.081	0.409
18	-0.464	0.315	0.245	0.181
19	-0.415	0.093	0.174	0.164
20	-0.602	-0.091	0.191	0.037
21	-0.561	0.271	0.146	-0.090
22	-0.595	-0.081	0.193	0.038
23	-0.669	-0.001	0.215	-0.090
24	-0.654	0.237	-0.112	0.055

UNIQUE VARIANCES

0.438	0.780	0.644	0.651	0.352	0.312	0.283	0.485	0.257	0.240
0.551	0.435	0.491	0.646	0.696	0.549	0.598	0.593	0.762	0.592
0.583	0.601	0.497	0.500						

VARIMAX-ROTATED FACTOR LOADINGS

	1	2	3	4
1	0.160	0.197	0.689	0.160
2	0.117	0.033	0.436	0.096
3	0.137	-0.019	0.570	0.110
4	0.233	0.099	0.527	0.080
5	0.739	0.213	0.185	0.150
6	0.767	0.066	0.205	0.233
7	0.806	0.153	0.197	0.075
8	0.569	0.242	0.338	0.132
9	0.806	0.040	0.201	0.227
10	0.168	0.831	-0.118	0.167
11	0.180	0.512	0.120	0.374
12	0.019	0.716	0.219	0.088
13	0.198	0.525	0.438	0.082
14	0.197	0.081	0.050	0.553
15	0.122	0.074	0.116	0.520
16	0.059	0.062	0.408	0.525
17	0.142	0.219	0.062	0.574
18	0.026	0.336	0.293	0.456
19	0.148	0.161	0.239	0.365
20	0.378	0.113	0.402	0.301
21	0.175	0.438	0.381	0.223
22	0.366	0.122	0.399	0.301
23	0.369	0.244	0.500	0.239
24	0.370	0.495	0.157	0.304

CHISQUARE WITH 185 DEGREES OF FREEDOM IS 226.6835

PROBABILITY LEVEL IS 0.022

TUCKER'S RELIABILITY COEFFICIENT= 0.952

	LATENT ROOTS	FIRST DIFFERENCES
1	0.2414D 01	- -
2	0.2177D 01	0.2365D 00
3	0.1832D 01	0.3451D 00
4	0.1700D 01	0.1324D 00
5	0.1551D 01	0.1486D 00
6	0.1467D 01	0.8450D-01
7	0.1385D 01	0.8138D-01
8	0.1292D 01	0.9345D-01
9	0.1161D 01	0.1309D 00
10	0.1091D 01	0.6969D-01
11	0.1054D 01	0.3691D-01
12	0.9810D 00	0.7336D-01
13	0.9003D 00	0.8066D-01
14	0.8404D 00	0.5988D-01
15	0.8033D 00	0.3713D-01
16	0.6883D 00	0.1150D 00
17	0.6751D 00	0.1325D-01
18	0.6417D 00	0.3342D-01
19	0.6224D 00	0.1932D-01
20	0.5620D 00	0.6035D-01
21	0.3861D 00	0.1759D 00
22	0.2546D 00	0.1315D 00
23	0.1706D 00	0.8401D-01
24	0.5347D-01	0.1171D 00

RESIDUAL CORRELATIONS

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	-0.055	1.000								
3	-0.049	0.061	1.000							
4	0.067	-0.061	-0.053	1.000						
5	0.028	0.163	0.059	-0.159	1.000					
6	0.058	0.055	0.050	0.033	-0.095	1.000				
7	-0.004	-0.092	-0.011	0.051	-0.063	0.122	1.000			
8	-0.128	-0.146	0.181	0.080	0.057	-0.066	0.126	1.000		
9	0.042	-0.028	-0.160	0.021	0.158	-0.003	-0.102	-0.098	1.000	
10	-0.035	0.009	-0.083	0.068	0.025	0.018	-0.020	0.020	-0.051	1.000
11	0.083	-0.003	-0.056	-0.127	0.054	0.168	-0.108	-0.031	0.015	-0.055
12	0.041	-0.029	0.041	-0.062	-0.009	-0.083	0.024	0.009	0.019	-0.009
13	0.099	-0.041	0.084	-0.011	0.001	0.056	0.057	0.005	0.003	-0.063
14	-0.084	-0.003	0.097	-0.090	0.052	-0.009	0.031	0.054	-0.093	-0.038
15	0.074	0.013	-0.110	-0.017	0.048	0.017	0.002	-0.035	0.013	-0.002
16	0.053	0.046	-0.060	0.071	-0.071	0.069	-0.011	0.067	-0.045	0.059
17	-0.044	-0.164	0.102	0.086	-0.090	0.007	0.060	0.046	0.020	-0.014
18	0.050	0.077	-0.005	0.033	0.109	-0.096	-0.013	-0.010	0.029	-0.021
19	-0.013	-0.076	0.169	-0.098	-0.102	-0.015	0.016	0.035	0.039	-0.003
20	-0.094	0.050	-0.023	0.005	-0.056	-0.034	0.066	0.015	-0.031	0.024
21	-0.077	0.091	-0.150	0.075	-0.018	-0.070	0.036	-0.003	-0.051	0.052
22	0.016	-0.035	-0.094	0.077	0.055	-0.127	-0.046	-0.103	0.087	-0.016
23	-0.030	0.070	0.045	-0.102	-0.042	-0.066	-0.122	0.064	0.118	0.050
24	-0.058	0.046	0.069	0.009	-0.080	0.034	0.025	-0.073	0.015	0.071

RESIDUAL CORRELATIONS

	11	12	13	14	15	16	17	18	19	20
11	1.000									
12	-0.001	1.000								
13	0.287	0.072	1.000							
14	0.100	0.018	0.086	1.000						
15	-0.046	0.086	-0.028	0.070	1.000					
16	0.044	-0.121	0.027	0.139	-0.014	1.000				
17	0.003	0.107	-0.040	-0.041	0.009	-0.046	1.000			
18	-0.057	0.012	-0.045	-0.159	0.054	-0.066	0.152	1.000		
19	0.024	-0.158	0.026	-0.072	-0.076	-0.080	0.064	0.094	1.000	
20	-0.153	0.035	-0.171	0.051	0.022	0.058	-0.026	-0.006	-0.170	1.000
21	0.025	-0.123	-0.043	-0.048	0.029	0.064	-0.168	-0.014	0.093	0.128
22	0.026	-0.024	-0.098	-0.041	-0.016	-0.123	-0.006	0.020	0.092	0.100
23	-0.171	0.091	-0.109	-0.013	0.018	-0.019	0.024	-0.138	0.004	0.126
24	-0.079	-0.018	-0.133	0.026	-0.158	-0.035	-0.036	0.081	0.147	0.024

RESIDUAL CORRELATIONS

	21	22	23	24
21	1.000			
22	0.063	1.000		
23	0.067	0.122	1.000	
24	0.071	0.045	0.051	1.000

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INTERMEDIATE RESULTS

ITER=  1   F=     0.1667571D 01
           PSI=   0.6620334D 00  0.7921309D 00  0.7079916D 00  0.7275153D 00  0.5415505D 00  0.5392895D 00  0.5320143D 00
                  0.6251172D 00  0.5068205D 00  0.6141544D 00  0.6412151D 00  0.6441769D 00  0.6424172D 00  0.7581073D 00
                  0.7958673D 00  0.7154812D 00  0.7255243D 00  0.7066814D 00  0.7529648D 00  0.6931650D 00  0.6868768D 00
                  0.7022890D 00  0.6270131D 00  0.6512756D 00
ITER=  2   F=     0.1428113D 01
           PSI=   0.6717967D 00  0.8908633D 00  0.8142132D 00  0.8160220D 00  0.6019711D 00  0.5390143D 00  0.5305280D 00
                  0.7041999D 00  0.5015878D 00  0.5091742D 00  0.6533026D 00  0.6722283D 00  0.5512077D 00  0.8048613D 00
                  0.8411701D 00  0.7398412D 00  0.7746416D 00  0.7730750D 00  0.8902538D 00  0.7212128D 00  0.7482566D 00
                  0.7591393D 00  0.6680250D 00  0.7065961D 00
ITER=  3   F=     0.1417544D 01
           PSI=   0.6684439D 00  0.8823449D 00  0.8026569D 00  0.8061618D 00  0.5954850D 00  0.5351182D 00  0.5297919D 00
                  0.6982078D 00  0.5118883D 00  0.4745967D 00  0.6285293D 00  0.6659219D 00  0.5143388D 00  0.7981839D 00
                  0.8380082D 00  0.7382130D 00  0.7803701D 00  0.7711973D 00  0.8744592D 00  0.7207170D 00  0.7494467D 00
                  0.7597896D 00  0.6635243D 00  0.6929721D 00
ITER=  4   F=     0.1417120D 01
           PSI=   0.6700473D 00  0.8833870D 00  0.8001476D 00  0.8057029D 00  0.5970908D 00  0.5368706D 00  0.5270526D 00
                  0.6971800D 00  0.5113986D 00  0.4663849D 00  0.6229695D 00  0.6658065D 00  0.5070088D 00  0.7982652D 00
                  0.8395329D 00  0.7403719D 00  0.7824976D 00  0.7717201D 00  0.8740751D 00  0.7215445D 00  0.7504336D 00
                  0.7609720D 00  0.6647560D 00  0.6914023D 00
ITER=  5   F=     0.1417095D 01
           PSI=   0.6708209D 00  0.8836751D 00  0.7991812D 00  0.8054322D 00  0.5971713D 00  0.5368349D 00  0.5264315D 00
                  0.6966185D 00  0.5119117D 00  0.4635441D 00  0.6211629D 00  0.6662991D 00  0.5058117D 00  0.7991139D 00
                  0.8399439D 00  0.7416218D 00  0.7833358D 00  0.7717515D 00  0.8739033D 00  0.7217758D 00  0.7507885D 00
                  0.7613114D 00  0.6651971D 00  0.6911012D 00
ITER=  6   F=     0.1417095D 01
           PSI=   0.6708194D 00  0.8836763D 00  0.7991830D 00  0.8054319D 00  0.5971711D 00  0.5368366D 00  0.5264303D 00
                  0.6966179D 00  0.5119100D 00  0.4634918D 00  0.6211448D 00  0.6663068D 00  0.5058236D 00  0.7991179D 00
                  0.8399470D 00  0.7416247D 00  0.7833389D 00  0.7717529D 00  0.8739038D 00  0.7217766D 00  0.7507911D 00
                  0.7613108D 00  0.6651979D 00  0.6911071D 00
```

MAXIMUM LIKELIHOOD SOLUTION FOR 5 FACTORS

UNROTATED FACTOR LOADINGS

	1	2	3	4	5
1	0.560	0.020	0.462	-0.147	0.020
2	0.344	-0.034	0.288	-0.044	0.119
3	0.376	-0.131	0.433	-0.120	0.027
4	0.462	-0.099	0.298	-0.118	0.160
5	0.727	-0.253	-0.219	-0.059	-0.015
6	0.723	-0.379	-0.162	0.001	-0.141
7	0.722	-0.354	-0.240	-0.136	0.017
8	0.689	-0.153	-0.030	-0.105	0.066
9	0.726	-0.424	-0.171	0.025	-0.029
10	0.512	0.595	-0.389	0.039	0.128
11	0.574	0.380	-0.036	0.074	-0.366
12	0.476	0.546	-0.016	-0.119	0.131
13	0.626	0.346	0.191	-0.385	-0.217
14	0.403	0.013	0.051	0.369	-0.245
15	0.359	0.026	0.123	0.373	-0.104
16	0.454	0.032	0.386	0.288	-0.105
17	0.437	0.139	0.039	0.402	-0.113
18	0.471	0.265	0.218	0.253	0.031
19	0.417	0.063	0.155	0.177	-0.057
20	0.592	-0.152	0.172	0.153	0.228
21	0.573	0.234	0.144	0.020	0.177
22	0.587	-0.126	0.176	0.122	0.119
23	0.668	-0.054	0.213	0.025	0.251
24	0.658	0.186	-0.134	0.132	0.138

UNIQUE VARIANCES

0.450	0.781	0.639	0.649	0.357	0.288	0.277	0.485	0.262	0.215
0.386	0.444	0.256	0.639	0.706	0.550	0.614	0.596	0.764	0.521
0.564	0.580	0.442	0.478						

VARIMAX-ROTATED FACTOR LOADINGS

	1	2	3	4	5
1	0.160	0.136	0.658	0.182	0.201
2	0.113	0.074	0.435	0.107	0.003
3	0.134	-0.054	0.561	0.107	0.117
4	0.231	0.092	0.533	0.071	0.010
5	0.736	0.192	0.188	0.162	0.061
6	0.775	0.029	0.187	0.251	0.113
7	0.809	0.135	0.208	0.069	0.047
8	0.568	0.223	0.349	0.131	0.059
9	0.800	0.030	0.216	0.224	-0.001
10	0.175	0.844	-0.100	0.176	0.035
11	0.185	0.436	0.057	0.451	0.428
12	0.023	0.690	0.222	0.101	0.142
13	0.196	0.456	0.424	0.084	0.561
14	0.197	0.055	0.050	0.556	0.086
15	0.121	0.066	0.130	0.508	-0.026
16	0.067	0.037	0.400	0.529	0.066
17	0.145	0.208	0.078	0.562	-0.006
18	0.025	0.325	0.306	0.452	0.006
19	0.147	0.145	0.242	0.364	0.053
20	0.370	0.140	0.453	0.287	-0.189
21	0.179	0.439	0.403	0.230	-0.001
22	0.353	0.125	0.423	0.302	-0.078
23	0.360	0.257	0.549	0.223	-0.106
24	0.371	0.502	0.185	0.307	-0.057

```
CHISQUARE WITH  166 DEGREES OF FREEDOM IS        186.8203
PROBABILITY LEVEL IS  0.128
TUCKER'S RELIABILITY COEFFICIENT=  0.973
```

	LATENT ROOTS	FIRST DIFFERENCES
1	0.2318D 01	- -
2	0.2050D 01	0.2677D 00
3	0.1748D 01	0.3021D 00
4	0.1648D 01	0.1005D 00
5	0.1489D 01	0.1585D 00
6	0.1434D 01	0.5479D-01
7	0.1301D 01	0.1332D 00
8	0.1163D 01	0.1381D 00
9	0.1089D 01	0.7426D-01
10	0.1021D 01	0.6779D-01
11	0.9760D 00	0.4510D-01
12	0.9261D 00	0.4983D-01
13	0.8573D 00	0.6883D-01
14	0.8069D 00	0.5041D-01
15	0.7718D 00	0.3509D-01
16	0.6814D 00	0.9038D-01
17	0.6611D 00	0.2033D-01
18	0.6325D 00	0.2859D-01
19	0.6096D 00	0.2286D-01
20	0.4332D 00	0.1765D 00
21	0.3434D 00	0.8983D-01
22	0.2435D 00	0.9981D-01
23	0.1545D 00	0.8905D-01
24	0.4958D-01	0.1049D 00

RESIDUAL CORRELATIONS

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	-0.023	1.000								
3	-0.044	0.070	1.000							
4	0.098	-0.060	-0.045	1.000						
5	0.029	0.168	0.060	-0.151	1.000					
6	0.042	0.075	0.047	0.062	-0.113	1.000				
7	-0.009	-0.091	-0.019	0.046	-0.058	0.106	1.000			
8	-0.116	-0.145	0.181	0.078	0.065	-0.065	0.122	1.000		
9	0.032	-0.034	-0.164	0.016	0.169	-0.012	-0.098	-0.095	1.000	
10	-0.000	-0.001	-0.055	0.058	0.029	0.053	-0.015	0.021	-0.058	1.000
11	0.033	0.042	-0.082	-0.078	0.039	0.075	-0.121	-0.015	0.019	-0.019
12	0.054	-0.028	0.041	-0.066	-0.003	-0.072	0.025	0.011	0.010	-0.006
13	-0.028	-0.025	0.019	0.012	-0.026	-0.045	0.045	-0.012	0.030	-0.006
14	-0.122	-0.006	0.087	-0.080	0.041	-0.049	0.039	0.058	-0.085	-0.013
15	0.065	0.002	-0.108	-0.018	0.044	0.014	0.009	-0.033	0.016	0.003
16	0.051	0.047	-0.056	0.086	-0.078	0.057	-0.006	0.071	-0.047	0.074
17	-0.054	-0.174	0.104	0.084	-0.093	-0.000	0.068	0.047	0.023	-0.009
18	0.067	0.068	0.006	0.032	0.111	-0.081	-0.002	-0.006	0.027	-0.030
19	-0.015	-0.077	0.171	-0.093	-0.103	-0.021	0.020	0.038	0.041	-0.001
20	-0.052	0.020	-0.014	-0.033	-0.048	0.023	0.073	0.003	-0.044	-0.025
21	-0.048	0.082	-0.141	0.064	-0.009	-0.036	0.043	-0.005	-0.057	0.013
22	0.041	-0.050	-0.086	0.064	0.066	-0.100	-0.039	-0.107	0.087	-0.049
23	0.002	0.044	0.054	-0.148	-0.030	-0.006	-0.131	0.054	0.117	0.005
24	-0.026	0.031	0.090	-0.007	-0.074	0.069	0.031	-0.076	0.008	0.026

RESIDUAL CORRELATIONS

	11	12	13	14	15	16	17	18	19	20
11	1.000									
12	0.009	1.000								
13	-0.000	0.033	1.000							
14	-0.002	0.017	0.043	1.000						
15	-0.071	0.085	0.006	0.083	1.000					
16	-0.009	-0.121	0.001	0.130	-0.007	1.000				
17	-0.023	0.110	-0.013	-0.024	0.027	-0.037	1.000			
18	-0.042	0.019	-0.002	-0.144	0.062	-0.056	0.163	1.000		
19	-0.002	-0.157	0.015	-0.072	-0.068	-0.077	0.072	0.101	1.000	
20	-0.003	0.080	-0.004	0.096	0.014	0.070	-0.032	-0.037	-0.176	1.000
21	0.107	-0.128	0.010	-0.037	0.021	0.067	-0.177	-0.029	0.093	0.075
22	0.119	-0.031	-0.015	-0.028	-0.026	-0.124	-0.014	0.002	0.095	0.037
23	-0.033	0.080	0.018	0.028	0.015	-0.009	0.024	-0.173	0.008	0.016
24	-0.001	-0.011	-0.035	0.051	-0.164	-0.028	-0.037	0.070	0.154	-0.049

RESIDUAL CORRELATIONS

	21	22	23	24
21	1.000			
22	0.031	1.000		
23	0.010	0.067	1.000	
24	0.037	0.006	-0.010	1.000

```
HARMAN'S 24 PSYCHOLOGICAL TESTS          ULS
N=  145
P=   24
KL=     4
KU=     5
MAXIT=    30
LOGICAL VARIABLES= FF
INTEGER VARIABLES= 3100
EPS= 0.0005000
EPSE= 0.1000000
LOGICAL SCRATCH TAPE (DISK) NUMBER=  4
```

MATRIX TO 8E ANALYZED

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	0.318	1.000								
3	0.403	0.317	1.000							
4	0.468	0.230	0.305	1.000						
5	0.321	0.285	0.247	0.227	1.000					
6	0.335	0.234	0.268	0.327	0.622	1.000				
7	0.304	0.157	0.223	0.335	0.656	0.722	1.000			
8	0.332	0.157	0.382	0.391	0.578	0.527	0.619	1.000		
9	0.326	0.195	0.184	0.325	0.723	0.714	0.685	0.532	1.000	
10	0.116	0.057	-0.075	0.099	0.311	0.203	0.246	0.285	0.170	1.000
11	0.308	0.150	0.091	0.110	0.344	0.353	0.232	0.300	0.280	0.484
12	0.314	0.145	0.140	0.160	0.215	0.095	0.181	0.271	0.113	0.585
13	0.489	0.239	0.321	0.32[illegible]	0.344	0.309	0.345	0.395	0.280	0.408
14	0.125	0.103	0.177	0.066	0.280	0.292	0.236	0.252	0.260	0.172
15	0.238	0.131	0.065	0.127	0.229	0.251	0.172	0.175	0.248	0.154
16	0.414	0.272	0.263	0.322	0.187	0.291	0.180	0.296	0.242	0.124
17	0.176	0.005	0.177	0.187	0.208	0.273	0.228	0.255	0.274	0.289
18	0.368	0.255	0.211	0.251	0.263	0.167	0.159	0.250	0.208	0.317
19	0.270	0.112	0.312	0.137	0.190	0.251	0.226	0.274	0.274	0.190
20	0.365	0.292	0.297	0.339	0.398	0.435	0.451	0.427	0.446	0.173
21	0.369	0.306	0.165	0.349	0.318	0.263	0.314	0.362	0.266	0.405
22	0.413	0.232	0.250	0.380	0.441	0.386	0.396	0.357	0.483	0.160
23	0.474	0.348	0.383	0.335	0.435	0.431	0.405	0.501	0.504	0.262
24	0.282	0.211	0.203	0.248	0.420	0.433	0.437	0.388	0.424	0.531

MATRIX TD 8E ANALYZEO

	11	12	13	14	15	16	17	18	19	20
11	1.000									
12	0.428	1.000								
13	0.535	0.512	1.000							
14	0.350	0.131	0.195	1.000						
15	0.240	0.173	0.139	0.370	1.000					
16	0.314	0.119	0.281	0.412	0.325	1.000				
17	0.362	0.278	0.194	0.341	0.345	0.324	1.000			
18	0.350	0.349	0.323	0.201	0.334	0.344	0.448	1.000		
19	0.290	0.110	0.263	0.206	0.192	0.258	0.324	0.358	1.000	
20	0.202	0.246	0.241	0.302	0.272	0.388	0.262	0.301	0.167	1.000
21	0.399	0.355	0.425	0.183	0.232	0.348	0.173	0.357	0.331	0.413
22	0.304	0.193	0.279	0.243	0.246	0.283	0.273	0.317	0.342	0.463
23	0.251	0.350	0.382	0.242	0.256	0.360	0.287	0.272	0.303	0.509
24	0.412	0.414	0.358	0.304	0.165	0.262	0.326	0.405	0.374	0.366

MATRIX TO 8E ANALYZEO

	21	22	23	24
21	1.000			
22	0.374	1.000		
23	0.451	0.503	1.000	
24	0.448	0.375	0.434	1.000

UNWEIGHTED LEAST SQUARES SOLUTION FOR 4 FACTORS

UNROTATED FACTOR LOADINGS

	1	2	3	4
1	-0.598	0.029	-0.380	0.217
2	-0.372	-0.030	-0.261	0.149
3	-0.420	-0.118	-0.364	0.125
4	-0.484	-0.108	-0.260	0.190
5	-0.688	-0.298	0.275	0.037
6	-0.687	-0.399	0.198	-0.078
7	-0.678	-0.413	0.303	0.076
8	-0.675	-0.195	0.092	0.106
9	-0.697	-0.449	0.226	-0.079
10	-0.475	0.528	0.478	0.103
11	-0.556	0.356	0.164	-0.089
12	-0.470	0.501	0.143	0.244
13	-0.599	0.262	-0.015	0.289
14	-0.425	0.058	-0.006	-0.424
15	-0.391	0.095	-0.094	-0.370
16	-0.511	0.086	-0.349	-0.247
17	-0.466	0.207	0.006	-0.394
18	-0.518	0.318	-0.158	-0.143
19	-0.443	0.099	-0.102	-0.135
20	-0.616	-0.133	-0.137	-0.038
21	-0.594	0.212	-0.072	0.137
22	-0.611	-0.105	-0.121	-0.031
23	-0.689	-0.062	-0.148	0.103
24	-0.651	0.170	0.189	0.000

UNIQUE VARIANCES

0.450	0.770	0.662	0.650	0.361	0.324	0.271	0.487	0.256	0.257
0.530	0.448	0.489	0.636	0.693	0.549	0.586	0.585	0.765	0.583
0.578	0.600	0.488	0.512						

VARIMAX-ROTATED FACTOR LOADINGS

	1	2	3	4
1	0.150	0.197	0.682	0.153
2	0.113	0.082	0.452	0.076
3	0.147	-0.011	0.551	0.114
4	0.230	0.090	0.533	0.070
5	0.731	0.216	0.194	0.143
6	0.757	0.071	0.212	0.230
7	0.814	0.153	0.192	0.073
8	0.568	0.230	0.343	0.138
9	0.809	0.051	0.199	0.218
10	0.171	0.824	-0.106	0.156
11	0.176	0.539	0.105	0.371
12	0.024	0.709	0.200	0.091
13	0.179	0.542	0.422	0.080
14	0.206	0.083	0.049	0.559
15	0.119	0.077	0.116	0.523
16	0.072	0.057	0.419	0.517
17	0.139	0.223	0.063	0.584
18	0.021	0.342	0.306	0.451
19	0.146	0.178	0.245	0.349
20	0.381	0.104	0.419	0.292
21	0.179	0.429	0.404	0.206
22	0.367	0.131	0.406	0.288
23	0.375	0.231	0.518	0.223
24	0.365	0.487	0.187	0.287

	LATENT ROOTS	FIRST DIFFERENCES
1	0.7646D 01	- -
2	0.1690D 01	0.5956D 01
3	0.1218D 01	0.4719D 00
4	0.9157D 00	0.3021D 00
5	0.4033D 00	0.5124D 00
6	0.3560D 00	0.4730D-01
7	0.2792D 00	0.7680D-01
8	0.2656D 00	0.1361D-01
9	0.2348D 00	0.3081D-01
10	0.1367D 00	0.9805D-01
11	0.7917D-01	0.5753D-01
12	0.4465D-01	0.3452D-01
13	0.1313D-01	0.3152D-01
14	-0.2655D-01	0.3968D-01
15	-0.3890D-01	0.1235D-01
16	-0.7655D-01	0.3765D-01
17	-0.1009D 00	0.2432D-01
18	-0.1343D 00	0.3341D-01
19	-0.1714D 00	0.3709D-01
20	-0.1855D 00	0.1410D-01
21	-0.2051D 00	0.1959D-01
22	-0.2455D 00	0.4046D-01
23	-0.3018D 00	0.5629D-01
24	-0.3260D 00	0.2420D-01

RESIDUAL CORRELATIONS

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	-0.060	1.000								
3	-0.019	0.061	1.000							
4	0.077	-0.070	-0.045	1.000						
5	0.035	0.163	0.038	-0.152	1.000					
6	0.073	0.059	0.031	0.039	-0.060	1.000				
7	0.025	-0.088	-0.022	0.063	-0.063	0.125	1.000			
8	-0.116	-0.150	0.169	0.084	0.063	-0.061	0.123	1.000		
9	0.074	-0.016	-0.168	0.032	0.168	0.020	-0.134	-0.107	1.000	
10	-0.071	0.014	-0.122	0.076	0.021	0.004	-0.042	0.036	-0.093	1.000
11	0.096	0.016	-0.049	-0.104	0.059	0.179	-0.108	-0.022	0.023	-0.099
12	0.045	-0.023	0.044	-0.041	-0.019	-0.097	0.021	0.027	-0.008	0.011
13	0.117	-0.037	0.104	0.012	0.007	0.069	0.080	0.026	0.018	-0.105
14	-0.077	0.012	0.087	-0.084	0.046	-0.019	0.013	0.039	-0.105	-0.035
15	0.082	0.026	-0.112	-0.009	0.056	0.022	0.006	-0.039	0.024	0.003
16	0.054	0.046	-0.062	0.068	-0.075	0.059	-0.016	0.051	-0.042	0.075
17	-0.041	-0.152	0.092	0.098	-0.082	0.009	0.064	0.042	0.025	-0.008
18	0.038	0.077	-0.014	0.033	0.109	-0.095	-0.006	-0.015	0.036	-0.016
19	-0.013	-0.074	0.165	-0.096	-0.100	-0.008	0.016	0.029	0.049	-0.022
20	-0.085	0.043	-0.036	-0.003	-0.056	-0.038	0.058	0.005	-0.038	0.053
21	-0.098	0.078	-0.166	0.064	-0.029	-0.082	0.025	-0.011	-0.067	0.082
22	0.022	-0.038	-0.094	0.076	0.052	-0.121	-0.055	-0.113	0.091	-0.034
23	-0.033	0.058	0.034	-0.113	-0.050	-0.076	-0.141	0.054	0.105	0.078
24	-0.084	0.037	0.033	0.001	-0.067	0.041	0.022	-0.071	0.012	0.116

RESIDUAL CORRELATIONS

	11	12	13	14	15	16	17	18	19	20
11	1.000									
12	-0.027	1.000								
13	0.269	0.067	1.000							
14	0.097	0.012	0.085	1.000						
15	-0.047	0.081	-0.026	0.062	1.000					
16	0.064	-0.109	0.036	0.141	-0.012	1.000				
17	-0.011	0.099	-0.047	-0.059	-0.003	-0.047	1.000			
18	-0.067	0.008	-0.059	-0.162	0.053	-0.068	0.147	1.000		
19	0.021	-0.171	0.015	-0.066	-0.069	-0.071	0.067	0.092	1.000	
20	-0.132	0.103	-0.157	0.051	0.027	0.049	-0.019	-0.005	-0.167	1.000
21	0.031	-0.105	-0.051	-0.040	0.036	0.062	-0.160	-0.018	0.087	0.122
22	0.034	-0.031	-0.096	-0.039	-0.009	-0.121	-0.002	0.018	0.096	0.094
23	-0.150	0.114	-0.095	-0.008	0.029	-0.025	0.038	-0.139	0.004	0.112
24	-0.078	-0.008	-0.147	0.033	-0.147	-0.036	-0.024	0.080	0.141	0.026

RESIDUAL CORRELATIONS

	21	22	23	24
21	1.000			
22	0.049	1.000		
23	0.056	0.112	1.000	
24	0.072	0.033	0.048	1.000

UNWEIGHTED LEAST SQUARES SOLUTION FOR 5 FACTORS

UNROTATED FACTOR LOADINGS

	1	2	3	4	5
1	-0.598	-0.024	-0.381	0.210	0.085
2	-0.372	0.035	-0.261	0.136	-0.071
3	-0.420	0.125	-0.366	0.128	0.117
4	-0.483	0.112	-0.258	0.183	-0.060
5	-0.686	0.293	0.279	0.044	0.027
6	-0.687	0.402	0.212	-0.063	0.140
7	-0.677	0.407	0.310	0.089	0.029
8	-0.673	0.194	0.094	0.109	0.037
9	-0.695	0.445	0.235	-0.066	-0.000
10	-0.477	-0.541	0.476	0.085	-0.170
11	-0.562	-0.376	0.168	-0.093	0.271
12	-0.468	-0.494	0.125	0.219	-0.065
13	-0.618	-0.306	-0.028	0.371	0.378
14	-0.425	-0.051	-0.003	-0.422	0.146
15	-0.390	-0.084	-0.092	-0.369	-0.003
16	-0.510	-0.075	-0.346	-0.251	0.074
17	-0.464	-0.195	0.004	-0.394	0.026
18	-0.518	-0.307	-0.165	-0.160	-0.081
19	-0.442	-0.093	-0.102	-0.136	0.030
20	-0.619	0.147	-0.141	-0.055	-0.225
21	-0.596	-0.210	-0.080	0.119	-0.180
22	-0.611	0.113	-0.120	-0.042	-0.141
23	-0.691	0.071	-0.152	0.092	-0.182
24	-0.652	-0.168	0.187	-0.017	-0.158

UNIQUE VARIANCES

0.445	0.769	0.644	0.651	0.362	0.298	0.272	0.487	0.260	0.217
0.431	0.469	0.242	0.617	0.696	0.546	0.591	0.579	0.766	0.522
0.548	0.578	0.452	0.487						

VARIMAX-ROTATED FACTOR LOADINGS

	1	2	3	4	5
1	0.155	0.147	0.666	0.168	0.192
2	0.111	0.079	0.455	0.075	-0.006
3	0.151	-0.059	0.537	0.126	0.159
4	0.228	0.082	0.535	0.068	0.013
5	0.731	0.198	0.197	0.148	0.052
6	0.769	0.034	0.199	0.241	0.112
7	0.815	0.138	0.195	0.075	0.044
8	0.571	0.203	0.342	0.146	0.093
9	0.805	0.050	0.208	0.213	-0.028
10	0.173	0.849	-0.088	0.157	0.002
11	0.188	0.477	0.071	0.419	0.353
12	0.030	0.678	0.208	0.106	0.124
13	0.186	0.476	0.415	0.086	0.563
14	0.210	0.053	0.042	0.573	0.081
15	0.118	0.078	0.127	0.516	-0.044
16	0.074	0.031	0.414	0.521	0.066
17	0.141	0.213	0.073	0.582	0.003
18	0.019	0.340	0.320	0.450	-0.015
19	0.147	0.161	0.246	0.351	0.051
20	0.373	0.135	0.455	0.275	-0.195
21	0.173	0.445	0.428	0.198	-0.048
22	0.362	0.147	0.426	0.277	-0.106
23	0.369	0.246	0.546	0.209	-0.098
24	0.363	0.503	0.209	0.284	-0.063

	LATENT ROOTS	FIRST DIFFERENCES
1	0.7674D 01	- -
2	0.1717D 01	0.5957D 01
3	0.1233D 01	0.4837D 00
4	0.9487D 00	0.2847D 00
5	0.4981D 00	0.4506D 00
6	0.3678D 00	0.1302D 00
7	0.2958D 00	0.7207D-01
8	0.2727D 00	0.2309D-01
9	0.2407D 00	0.3195D-01
10	0.1577D 00	0.8302D-01
11	0.9655D-01	0.6118D-01
12	0.6441D-01	0.3214D-01
13	0.3177D-01	0.3264D-01
14	0.8511D-02	0.2326D-01
15	-0.3545D-02	0.1206D-01
16	-0.3796D-01	0.3441D-01
17	-0.7134D-01	0.3339D-01
18	-0.1037D 00	0.3239D-01
19	-0.1436D 00	0.3989D-01
20	-0.1644D 00	0.2078D-01
21	-0.1871D 00	0.2271D-01
22	-0.2263D 00	0.3923D-01
23	-0.2838D 00	0.5744D-01
24	-0.3141D 00	0.3035D-01

RESIDUAL CORRELATIONS

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	-0.043	1.000								
3	-0.040	0.074	1.000							
4	0.094	-0.070	-0.035	1.000						
5	0.031	0.167	0.033	-0.148	1.000					
6	0.044	0.080	-0.003	0.056	-0.084	1.000				
7	0.018	-0.083	-0.031	0.066	-0.060	0.103	1.000			
8	-0.121	-0.143	0.163	0.090	0.064	-0.084	0.122	1.000		
9	0.073	-0.019	-0.168	0.029	0.174	0.014	-0.124	-0.104	1.000	
10	-0.012	-0.002	-0.064	0.069	0.035	0.080	-0.029	0.065	-0.113	1.000
11	0.053	0.052	-0.107	-0.081	0.045	0.106	-0.139	-0.047	0.034	-0.046
12	0.064	-0.023	0.055	-0.041	-0.011	-0.086	0.025	0.038	-0.022	0.017
13	-0.027	-0.025	-0.006	0.026	-0.033	-0.062	0.062	-0.040	0.071	-0.027
14	-0.105	0.020	0.065	-0.077	0.039	-0.061	0.007	0.030	-0.099	0.010
15	0.082	0.021	-0.111	-0.013	0.056	0.030	0.009	-0.038	0.028	-0.006
16	0.044	0.053	-0.076	0.076	-0.080	0.044	-0.019	0.047	-0.038	0.113
17	-0.047	-0.154	0.090	0.095	-0.082	0.006	0.064	0.040	0.029	-0.005
18	0.057	0.070	0.003	0.028	0.115	-0.071	0.002	-0.006	0.034	-0.049
19	-0.016	-0.072	0.163	-0.093	-0.100	-0.014	0.016	0.029	0.051	-0.014
20	-0.051	0.018	0.001	-0.028	-0.051	0.021	0.073	0.018	-0.054	-0.027
21	-0.066	0.065	-0.138	0.053	-0.017	-0.031	0.040	0.005	-0.074	0.012
22	0.050	-0.052	-0.070	0.066	0.061	-0.088	-0.046	-0.104	0.087	-0.094
23	0.001	0.040	0.070	-0.136	-0.042	-0.025	-0.135	0.070	0.099	0.015
24	-0.049	0.024	0.071	-0.011	-0.060	0.089	0.034	-0.058	0.002	0.046

RESIDUAL CORRELATIONS

	11	12	13	14	15	16	17	18	19	20
11	1.000									
12	-0.010	1.000								
13	0.027	0.054	1.000							
14	0.026	0.016	0.046	1.000						
15	-0.053	0.072	0.019	0.067	1.000					
16	0.029	-0.106	-0.004	0.127	-0.007	1.000				
17	-0.033	0.098	-0.042	-0.061	0.004	-0.047	1.000			
18	-0.043	0.010	-0.015	-0.152	0.050	-0.060	0.150	1.000		
19	0.005	-0.164	-0.005	-0.071	-0.066	-0.071	0.069	0.097	1.000	
20	-0.023	0.088	0.014	0.098	0.015	0.070	-0.021	-0.044	-0.171	1.000
21	0.119	-0.110	0.037	-0.008	0.029	0.080	-0.163	-0.044	0.095	0.056
22	0.115	-0.043	0.003	-0.014	-0.015	-0.110	-0.002	-0.004	0.103	0.032
23	-0.062	0.105	0.021	0.031	0.021	-0.007	0.041	-0.174	0.010	0.027
24	-0.017	-0.008	-0.072	0.063	-0.159	-0.021	-0.023	0.060	0.150	-0.045

RESIDUAL CORRELATIONS

	21	22	23	24
21	1.000			
22	0.007	1.000		
23	-0.004	0.063	1.000	
24	0.025	-0.009	-0.007	1.000

```
HARMAN'S 13 PSYCHOLOGICAL TESTS           ML

N=  145

P=   24

KL=    4

KU=    4

MAXIT=   30

LOGICAL VARIABLES= TF

INTEGER VARIABLES= 3300

EPS= 0.0005000

EPSE= 0.1000000

LOGICAL SCRATCH TAPE (DISK) NUMBER=  4
```

MATRIX TO BE ANALYZED

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	0.318	1.000								
3	0.403	0.317	1.000							
4	0.468	0.230	0.305	1.000						
5	0.321	0.285	0.247	0.227	1.000					
6	0.335	0.234	0.268	0.327	0.622	1.000				
7	0.304	0.157	0.223	0.335	0.656	0.722	1.000			
8	0.332	0.157	0.382	0.391	0.578	0.527	0.619	1.000		
9	0.326	0.195	0.184	0.325	0.723	0.714	0.685	0.532	1.000	
10	0.116	0.057	-0.075	0.099	0.311	0.203	0.246	0.285	0.170	1.000
11	0.308	0.150	0.091	0.110	0.344	0.353	0.232	0.300	0.280	0.484
12	0.314	0.145	0.140	0.160	0.215	0.095	0.181	0.271	0.113	0.585
13	0.489	0.239	0.321	0.327	0.344	0.309	0.345	0.395	0.280	0.408

MATRIX TO BE ANALYZED

	11	12	13
11	1.000		
12	0.428	1.000	
13	0.535	0.512	1.000

F21P4000-1.N 348005

MAXIMUM LIKELIHOOD SOLUTION FOR 4 FACTORS

UNROTATED FACTOR LOADINGS

	1	2	3	4
1	0.309	0.395	-0.236	-0.482
2	0.151	0.262	-0.097	-0.282
3	0.092	0.359	-0.128	-0.502
4	0.111	0.443	-0.179	-0.325
5	0.346	0.716	0.044	0.130
6	0.355	0.734	0.211	0.037
7	0.234	0.806	0.048	0.129
8	0.302	0.646	-0.114	-0.026
9	0.282	0.773	0.198	0.093
10	0.485	0.141	-0.507	0.425
11	1.000	-0.002	0.001	-0.000
12	0.429	0.124	-0.647	0.071
13	0.536	0.300	-0.399	-0.216

UNIQUE VARIANCES

0.461 0.820 0.595 0.654 0.349 0.289 0.277 0.478 0.276 0.307
0.000 0.378 0.417

VARIMAX-ROTATED FACTOR LOADINGS

	1	2	3	4
1	0.140	0.186	0.154	0.679
2	0.061	0.141	0.046	0.393
3	-0.002	0.135	-0.032	0.621
4	-0.048	0.254	0.090	0.521
5	0.105	0.753	0.185	0.197
6	0.177	0.791	0.015	0.233
7	-0.018	0.809	0.142	0.217
8	0.050	0.586	0.236	0.347
9	0.090	0.823	0.020	0.194
10	0.162	0.209	0.784	-0.091
11	0.887	0.204	0.394	0.126
12	0.112	0.028	0.742	0.243
13	0.279	0.186	0.476	0.494

CHISQUARE WITH 32 DEGREES OF FREEDOM IS 47.4241

PROBABILITY LEVEL IS 0.039

TUCKER'S RELIABILITY COEFFICIENT= 0.950

	LATENT ROOTS	FIRST DIFFERENCES
1	0.1669D 01	- -
2	0.1554D 01	0.1152D 00
3	0.1181D 01	0.3730D 00
4	0.1125D 01	0.5573D-01
5	0.1008D 01	0.1171D 00
6	0.8990D 00	0.1088D 00
7	0.8633D 00	0.3576D-01
8	0.7399D 00	0.1234D 00
9	0.7109D 00	0.2898D-01
10	0.3284D 00	0.3825D 00
11	0.2582D 00	0.7023D-01
12	0.9104D-01	0.1672D 00
13	0.4991D-03	0.9055D-01

RESIDUAL CORRELATIONS

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	0.015	1.000								
3	-0.074	0.079	1.000							
4	0.109	-0.016	-0.080	1.000						
5	0.010	0.161	0.064	-0.164	1.000					
6	0.007	0.039	0.042	0.028	-0.128	1.000				
7	-0.037	-0.101	-0.041	0.007	-0.066	0.115	1.000			
8	-0.119	-0.121	0.178	0.076	0.048	-0.079	0.102	1.000		
9	0.071	-0.009	-0.116	0.041	0.166	0.005	-0.090	-0.075	1.000	
10	-0.012	0.035	-0.051	0.068	0.029	0.053	-0.040	0.003	-0.051	1.000
11	-0.000	-0.001	-0.000	-0.000	-0.000	0.001	-0.001	0.002	-0.001	-0.000
12	0.033	0.009	0.019	-0.071	-0.007	-0.043	0.008	-0.024	0.055	0.005
13	0.015	-0.034	0.010	-0.013	-0.027	-0.026	0.074	-0.025	-0.010	-0.012

RESIDUAL CORRELATIONS

	11	12	13
11	1.000		
12	-0.000	1.000	
13	0.001	0.006	1.000

HARMAN'S 12 PSYCHOLOGICAL TESTS ML

N= 145

P= 24

KL= 3

KU= 4

MAXIT= 30

LOGICAL VARIABLES= TF

INTEGER VARIABLES= 3300

EPS= 0.0005000

EPSE= 0.1000000

LOGICAL SCRATCH TAPE (DISK) NUMBER= 4

MATRIX TO BE ANALYZED

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	0.408	1.000								
3	0.512	0.585	1.000							
4	0.344	0.311	0.215	1.000						
5	0.327	0.099	0.160	0.227	1.000					
6	0.280	0.170	0.113	0.723	0.325	1.000				
7	0.489	0.116	0.314	0.321	0.468	0.326	1.000			
8	0.395	0.285	0.271	0.578	0.391	0.532	0.332	1.000		
9	0.345	0.246	0.181	0.656	0.335	0.685	0.304	0.619	1.000	
10	0.239	0.057	0.145	0.285	0.230	0.195	0.318	0.157	0.157	1.000
11	0.309	0.203	0.095	0.622	0.327	0.714	0.335	0.527	0.722	0.234
12	0.321	-0.075	0.140	0.247	0.305	0.184	0.403	0.382	0.223	0.317

MATRIX TO BE ANALYZED

	11	12
11	1.000	
12	0.268	1.000

MAXIMUM LIKELIHOOD SOLUTION FOR 3 FACTORS

UNROTATED FACTOR LOADINGS

	1	2	3
1	0.550	0.416	-0.278
2	0.418	0.653	0.333
3	0.378	0.680	-0.057
4	0.793	-0.110	0.134
5	0.449	0.005	-0.354
6	0.795	-0.289	0.128
7	0.506	0.124	-0.517
8	0.714	-0.003	-0.054
9	0.807	-0.184	0.122
10	0.307	0.021	-0.294
11	0.786	-0.258	0.074
12	0.357	-0.049	-0.521

UNIQUE VARIANCES

0.448	0.281	0.391	0.342	0.673	0.267	0.461	0.487	0.300	0.819
0.310	0.598								

VARIMAX-ROTATED FACTOR LOADINGS

	1	2	3
1	0.199	0.510	0.503
2	0.212	0.816	-0.094
3	0.029	0.740	0.246
4	0.761	0.198	0.200
5	0.249	0.083	0.508
6	0.832	0.033	0.197
7	0.192	0.177	0.686
8	0.585	0.230	0.345
9	0.798	0.132	0.213
10	0.145	0.062	0.395
11	0.792	0.047	0.244
12	0.133	-0.033	0.619

CHISQUARE WITH 33 DEGREES OF FREEDOM IS 46.5319

PROBABILITY LEVEL IS 0.059

TUCKER'S RELIABILITY COEFFICIENT= 0.961

	LATENT ROOTS	FIRST DIFFERENCES
1	0.1667D 01	- -
2	0.1524D 01	0.1435D 00
3	0.1173D 01	0.3506D 00
4	0.1120D 01	0.5326D-01
5	0.1024D 01	0.9556D-01
6	0.9038D 00	0.1203D 00
7	0.8644D 00	0.3942D-01
8	0.7422D 00	0.1222D 00
9	0.7087D 00	0.3355D-01
10	0.3237D 00	0.3849D 00
11	0.2074D 00	0.1163D 00
12	0.7544D-01	0.1320D 00

RESIDUAL CORRELATIONS

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	-0.007	1.000								
3	0.013	-0.005	1.000							
4	-0.022	0.022	-0.007	1.000						
5	-0.037	0.060	-0.065	-0.168	1.000					
6	-0.004	-0.055	0.050	0.145	0.035	1.000				
7	0.034	-0.014	0.021	0.007	0.103	0.071	1.000			
8	-0.025	0.016	-0.001	0.046	0.090	-0.083	-0.121	1.000		
9	0.032	-0.039	0.022	-0.062	0.038	-0.090	-0.050	0.127	1.000	
10	-0.033	0.027	-0.004	0.158	-0.016	-0.012	0.013	-0.124	-0.103	1.000
11	0.013	0.066	-0.065	-0.120	0.004	0.017	0.018	-0.080	0.103	0.039
12	0.000	-0.045	0.018	0.063	-0.062	-0.120	-0.079	0.182	-0.026	0.079

RESIDUAL CORRELATIONS

	11	12
11	1.000	
12	0.029	1.000

MAXIMUM LIKELIHOOD SOLUTION FOR 4 FACTORS

UNROTATED FACTOR LOADINGS

	1	2	3	4
1	0.345	-0.614	0.132	0.193
2	0.312	-0.477	0.528	-0.313
3	0.216	-0.596	0.465	0.037
4	1.000	0.002	0.001	0.001
5	0.228	-0.443	-0.273	0.201
6	0.724	-0.180	-0.346	-0.170
7	0.322	-0.488	-0.151	0.418
8	0.579	-0.383	-0.177	-0.028
9	0.657	-0.335	-0.349	-0.259
10	0.285	-0.166	-0.058	0.322
11	0.623	-0.312	-0.431	-0.216
12	0.248	-0.285	-0.239	0.455

UNIQUE VARIANCES

0.449 0.299 0.381 0.000 0.636 0.295 0.460 0.486 0.267 0.784
0.281 0.594

VARIMAX-ROTATED FACTOR LOADINGS

	1	2	3	4
1	0.203	0.006	0.520	0.490
2	0.193	0.065	0.806	-0.099
3	0.016	0.022	0.750	0.237
4	0.705	0.653	0.189	0.202
5	0.286	-0.144	0.098	0.501
6	0.797	0.174	0.045	0.196
7	0.198	0.012	0.187	0.682
8	0.582	0.085	0.240	0.332
9	0.821	0.016	0.150	0.189
10	0.111	0.173	0.056	0.413
11	0.815	-0.015	0.061	0.227
12	0.134	0.050	-0.029	0.620

CHISQUARE WITH 24 DEGREES OF FREEDOM IS 28.9430

PROBABILITY LEVEL IS 0.222

TUCKER'S RELIABILITY COEFFICIENT= 0.980

	LATENT ROOTS	FIRST DIFFERENCES
1	0.1496D 01	0.1350D 00
2	0.1361D 01	0.2268D 00
3	0.1134D 01	0.8531D-01
4	0.1049D 01	0.6840D-01
5	0.9802D 00	0.1045D 00
6	0.8757D 00	0.4879D-01
7	0.8269D 00	0.1073D 00
8	0.7197D 00	0.3694D 00
9	0.3503D 00	0.1227D 00
10	0.2276D 00	0.5281D-01
11	0.1748D 00	0.1743D 00
12	0.4983D-03	

RESIDUAL CORRELATIONS

	1	2	3	4	5	6	7	8	9	10
1	1.000									
2	-0.006	1.000								
3	0.007	-0.001	1.000							
4	-0.000	0.001	-0.001	1.000						
5	-0.050	0.054	-0.068	-0.001	1.000					
6	-0.006	-0.042	0.048	0.001	0.045	1.000				
7	0.037	-0.019	0.020	0.001	0.097	0.064	1.000			
8	-0.024	0.016	0.003	0.001	0.084	-0.059	-0.119	1.000		
9	0.023	-0.056	0.034	0.001	-0.016	-0.057	-0.045	0.114	1.000	
10	-0.027	0.041	-0.001	0.001	0.015	-0.014	0.002	-0.118	-0.051	1.000
11	0.002	0.069	-0.052	-0.002	-0.066	0.071	0.020	-0.097	0.004	0.104
12	0.009	-0.047	0.023	0.000	-0.056	-0.124	-0.081	0.187	-0.002	0.057

RESIDUAL CORRELATIONS

	11	12
11	1.000	
12	0.049	1.000

Appendix C

If the user wishes to change the number of variables, p , and/or the number of variables before selection, p_0 , the MAIN program and subroutines REX, SELECT, NWTRAP, INCPSI and FCTGR need to be modified.

In the MAIN program the DIMENSIØN card should read as follows:

DIMENSIØN FMT(10),S(n),A(n),HEAD(20),YY(p),E(m),Y(p)

where $n = (p_0(p_0 + 1))/2$ and $m = (p(p + 1))/2$.

In subroutine REX the DIMENSIØN card should be:

DIMENSIØN S(1),E(1),Y(p_0),X(p_0),FMT(10) .

In subroutine SELECT the DIMENSIØN card should be:

DIMENSIØN S(1),E(1),MM(p_0) .

In subroutines NWTRAP, INCPSI and FCTGR the CØMMØN block KERN should read:

CØMMØN/KERN/G(p),V(p),VB(p),D2(p),S1(p),S2(p),S3(p),EPSU,
BND,IM(p),MØR,KP,MØRE,MAXTRY,P2,KP1 .

Caution: The following relationship between p and p_0 must hold

$$p^2 \leq (p_0(p_0 + 1))/2 \quad .$$